Permanent Loop Current
in Strongly Correlated Electron Systems
based on fRG

fRG in condensed matter physics

Rina Tazai (YITP, Kyoto university)

outline

1. Introduction: fRG in condensed matter physics

2. Recent study of "non-local phase transition" based on fRG

ex. permanent loop current

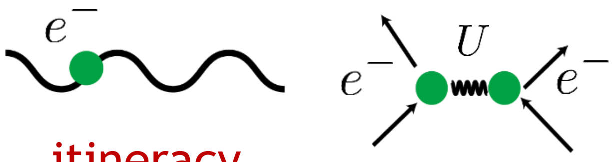
①coupled-chain Hubbrad model
②kagome superconductor (2019~)

RT et al., Phys. Rev. B 103, L161112 (2021).
RT *et al.*, Sci. Adv. **8**, eabl4108 (2022).

3. summary

Itineracy and locality of electron

electrons in metal



itineracy

locality

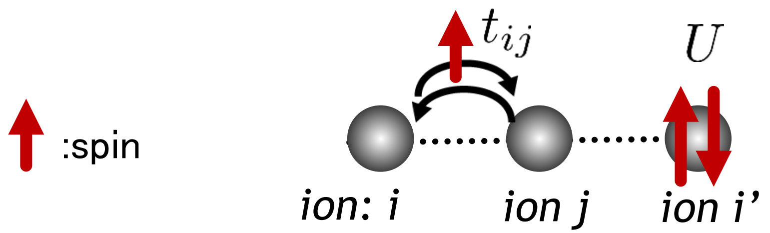
duality

Hubbard model

$$\hat{H} = \sum_{i,j} t_{ij} c_i^{\dagger} c_j + \sum_i U n_{i\uparrow} n_{i\downarrow}$$

hopping (itineracy)

Coulomb repulsion (locality)



intermediate region of itineracy and locality
(U≈t)

- magnetic order
- charge order
- superconductivity

fRG is powerful in intermediate region.

fRG in Hubbard model

- g-ology fRG → 1D
- parquet fRG (Shultz) → 2D
- N-patch fRG (Metzner)
- fRG for orbital-order
- fRG for loop current/bond-order

fRG in Hubbard model

$$\hat{H} = \sum_{i,j} t_{ij} c_i^\dagger c_j + \sum_i U n_{i\uparrow} n_{i\downarrow}$$

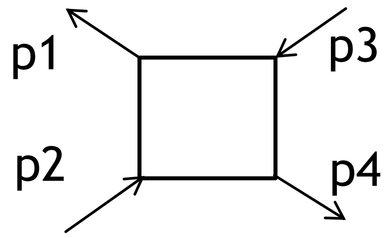
Fourier transformation

$$= \sum_\sigma \epsilon_k c^\dagger_{k+q,\sigma} c_{k,\sigma} + \frac{1}{4}\sum_{\{p_i\}} \underline{g_{p_1p_2p_3p_4}}\, c^\dagger_{p_1} c_{p_2} c_{p_3} c^\dagger_{p_4}$$

→ band

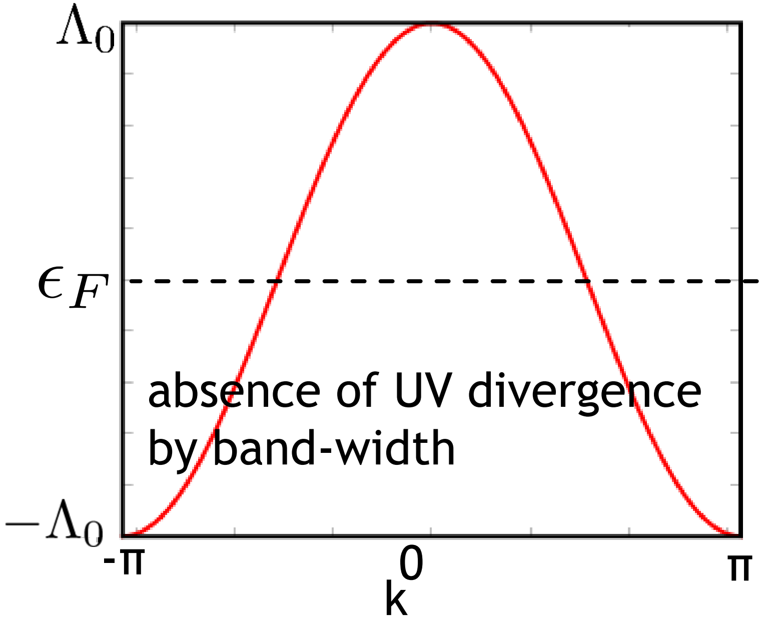
$p_i \equiv (k_i, \sigma_i)$

$g_{p_1p_2p_3p_4}$



$g_{p_1p_2p_3p_4} = U\delta_{p_1+p_4,p_2+p_3}(\delta_{\sigma_1,\sigma_3}\delta_{\sigma_2,\sigma_4} - \delta_{\sigma_1,\sigma_2}\delta_{\sigma_3,\sigma_4})$ = initial 4-point vertex

ϵ_k cos(k) band dispersion

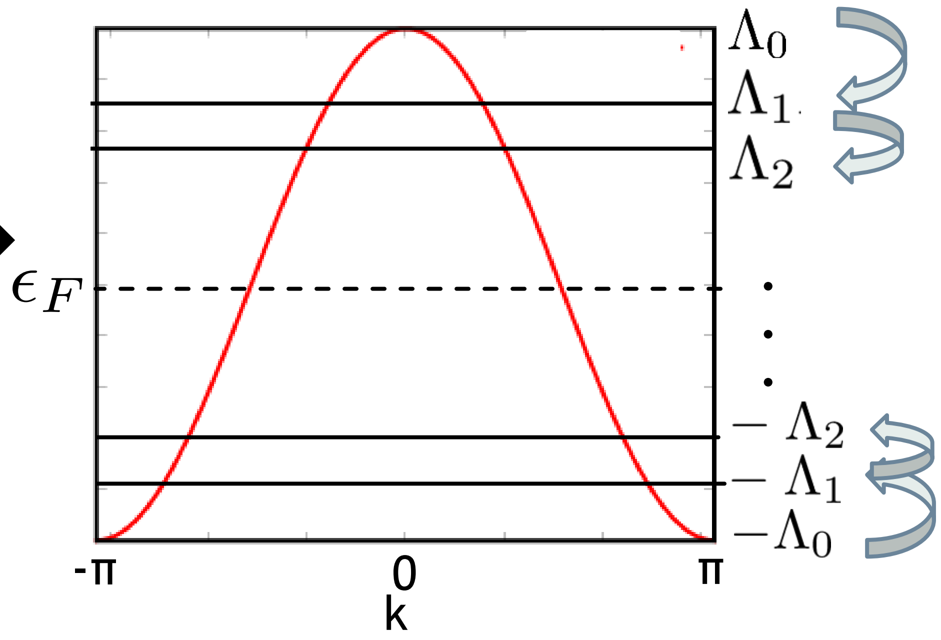


logarithmic energy cutoff

$$\Lambda_l = \Lambda_0 e^{-l\delta}$$

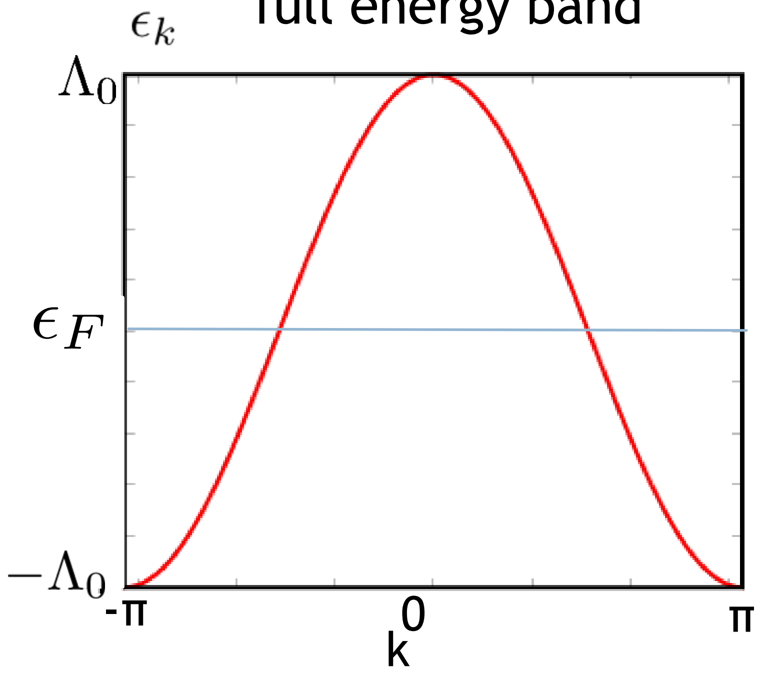
l=0,1,2,3・・・

δ: small value



fRG in Hubbard model

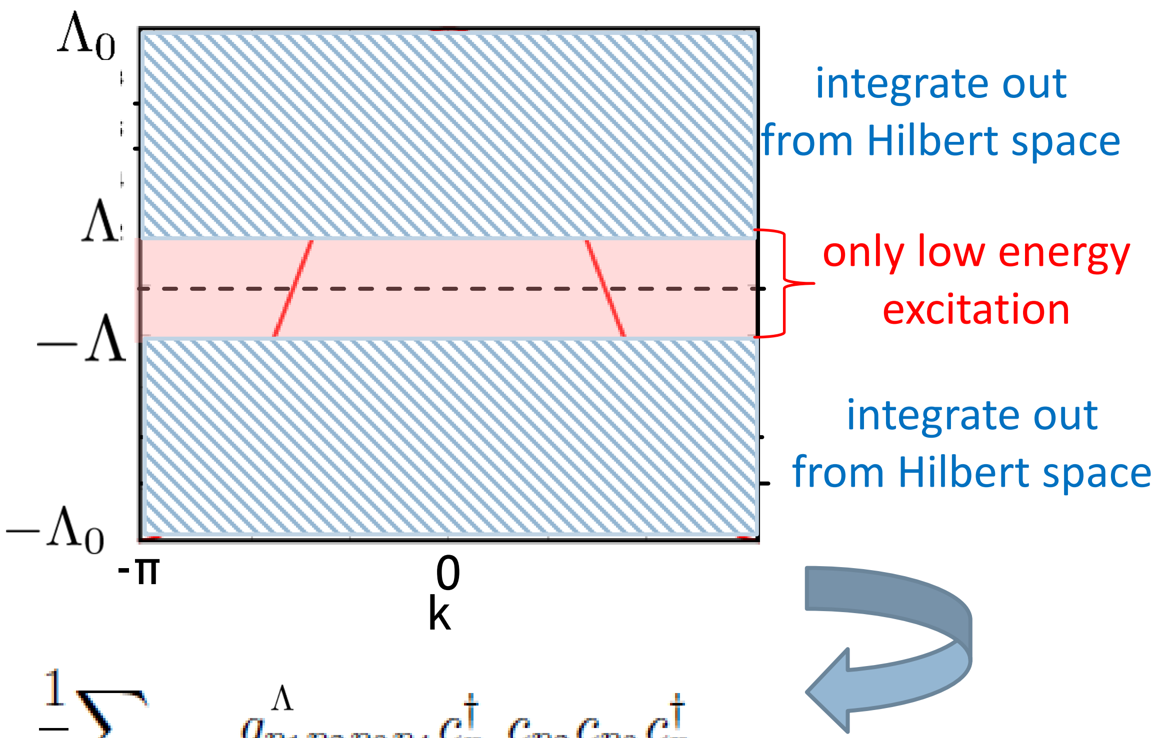
full energy band



$$\Lambda_l = \Lambda_0 e^{-l/10}$$

$$\Lambda_l : \Lambda_0 \to \Lambda$$

low energy effective model



$$\hat{H}_{eff} = \sum_\sigma \epsilon_k c^\dagger_{k+q,\sigma} c_{k,\sigma} + \frac{1}{4}\sum_{\{p_i\}} \underline{g^\Lambda_{p_1p_2p_3p_4}} c^\dagger_{p_1} c_{p_2} c_{p_3} c^\dagger_{p_4}$$

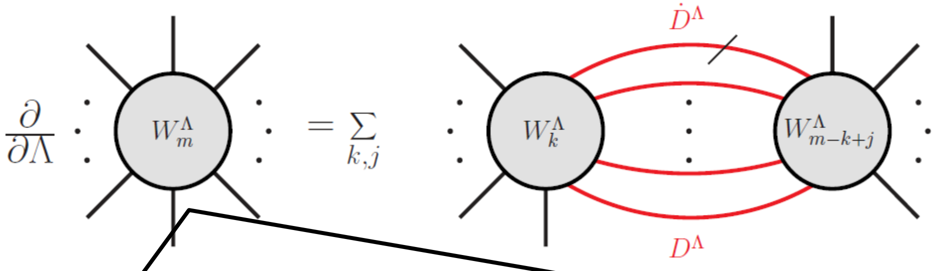
low energy effective interaction

fRG in Hubbard model

W. Metzner, et al., Rev. Mod. Phys. 84, 299 (2012).
T. Enss, PhD thesis (2005).

$$\hat{H}_{eff} \quad \frac{1}{4}\sum_{\{p_i\}} g^{\Lambda}_{p_1 p_2 p_3 p_4} c^{\dagger}_{p_1} c_{p_2} c_{p_3} c^{\dagger}_{p_4}$$

low energy effective interaction



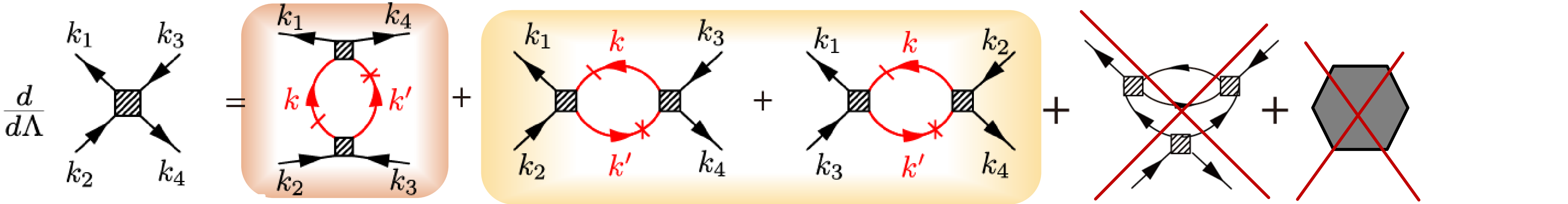
fRG differential equation

Wick-ordered fRG

$$W^{\pm}_{p,p'} = G^{\Lambda}_{p}(\epsilon)G^{\Lambda}_{p'}(\pm\epsilon)$$

$$G^{\Lambda}_{p}(\epsilon) = (i\epsilon - \epsilon_k)^{-1}\theta(\Lambda_l - |\epsilon_k|)$$

$$\frac{d}{d\Lambda_l} g_{p_1p_2p_3p_4} = \sum_{pp'}\left[\frac{1}{2}\frac{dW^{-}_{p,p'}}{d\Lambda_l} g_{p_1pp'p_4} g_{pp_2p_3p'} + \frac{dW^{+}_{p,p'}}{d\Lambda_l}\left(g_{p_1p_3pp'} g_{pp'p_2p_4} - g_{p_1p_2pp'} g_{pp'p_3p_4}\right)\right]$$



Cooper   Peierls   2-loop   6-point vertex

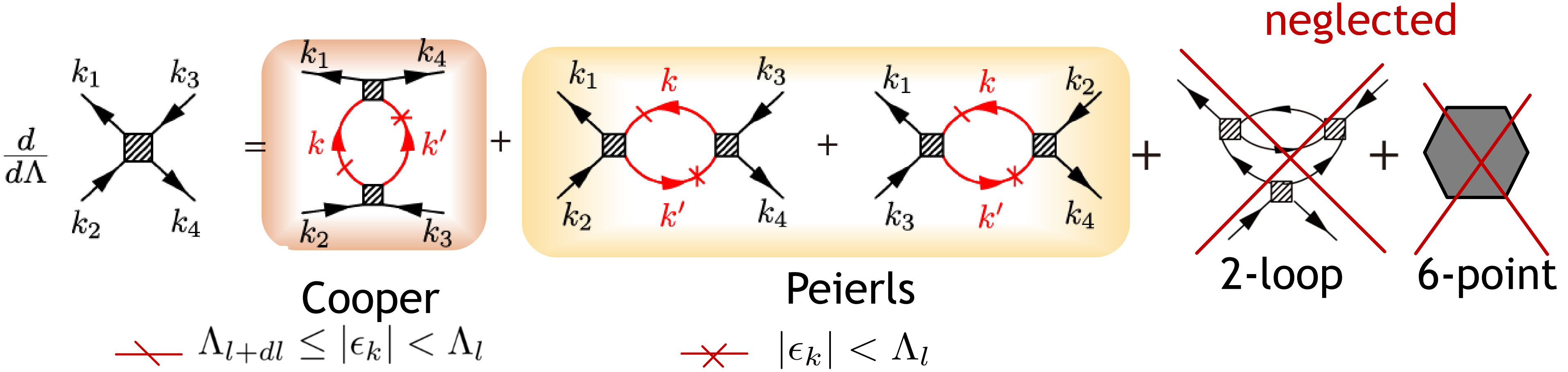
$\Lambda_{l+dl} \leq |\epsilon_k| < \Lambda_l$

on shell

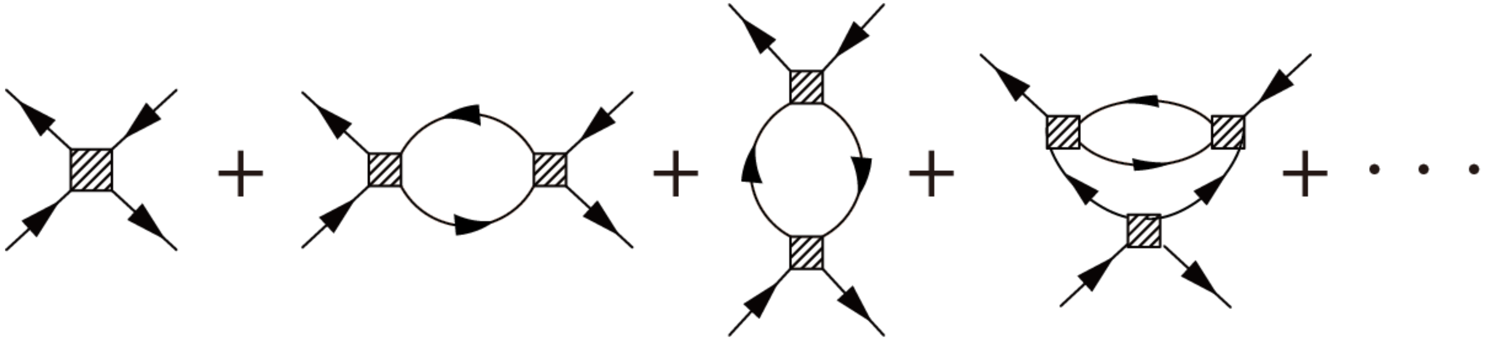
$|\epsilon_k| < \Lambda_l$

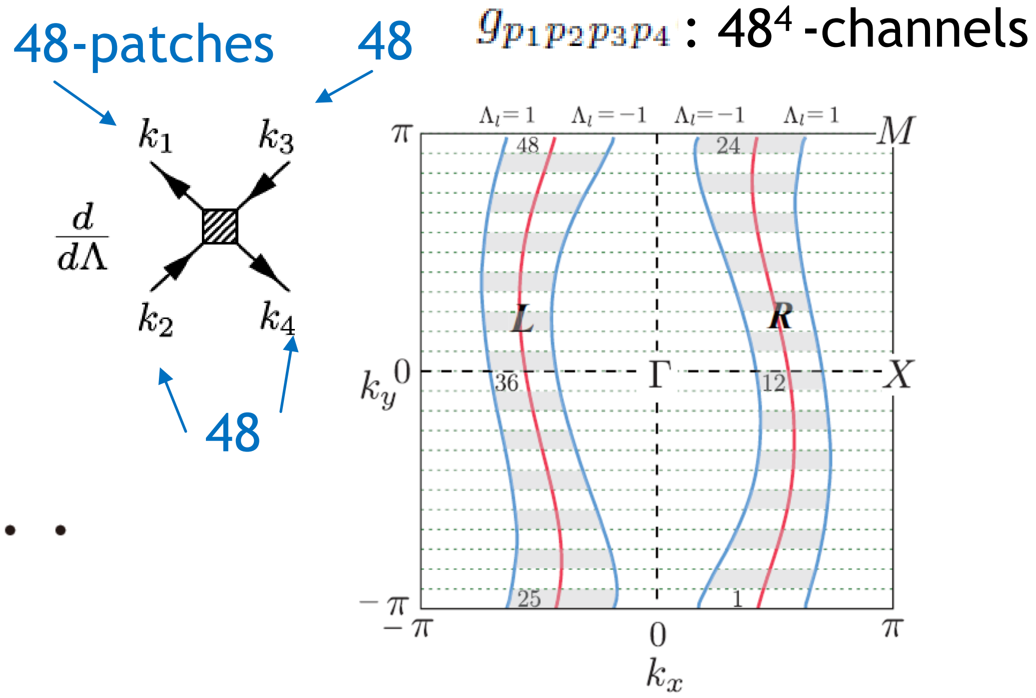
low energy

fRG in Hubbard model



Parquet diagrams are automatically generated.





advantage of fRG

✓ advantage

- ✓ Higher-order scatterings of U are automatically considered.
- ✓ Vertex-corrections are automatically considered.

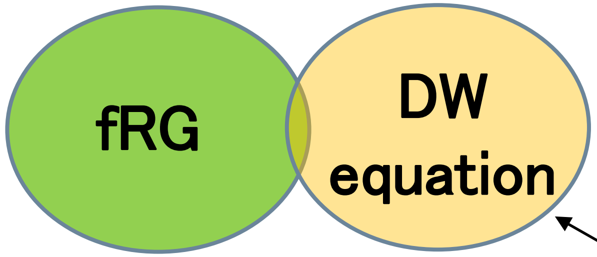
→ beyond RPA/mean-field

✕ disadvantage

- ✓ Physical meaning is unclear.

  ex. Which diagram/process are important ?

Both fRG & diagrammatic calculation are needed.



diagrammatic calculation with vertex corrections (later)

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3. summary

non-local phase transitions

recent discovery

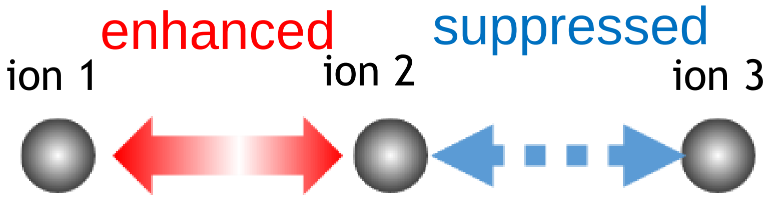
Based on fRG,
"new types of phase transitions" have been discovered!

= non-local order

order parameter of non−local order

$$\hat{O} = \sum_{i \neq j} \delta t_{ij} c_i^\dagger c_j$$

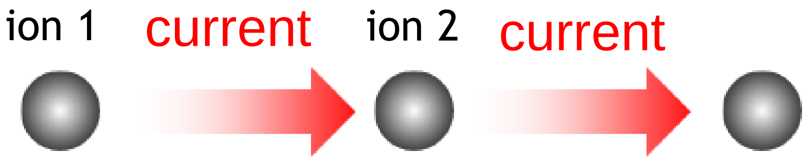
= spontaneous symmetry breaking of hopping



= bond order

$\delta t_{12} =$ +a $\delta t_{23} =$ -a

a: real #



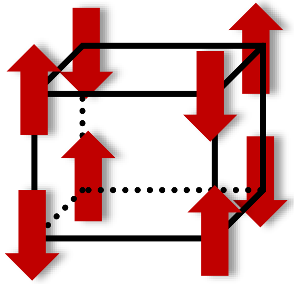
= loop current

$\delta t_{12} =$ +ia $\delta t_{23} =$ +ia

local VS non-local order

local order

ex. spin order, charge order



order parameter

$\langle S_i \rangle \neq 0$

i: lattice index

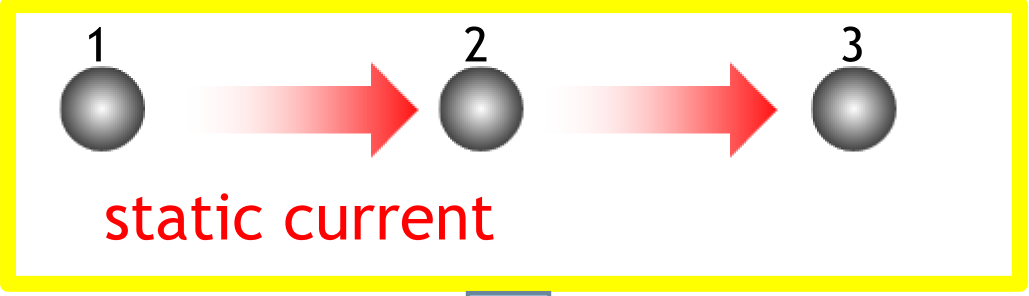
$$\hat{O} = \sum \hat{S}_z \, c^\dagger_{i\sigma} c_{i\sigma}$$

σ: spin

same site

non-local order

ex. loop current order



order parameter

$$\hat{O} = \sum_{i \neq j} \delta t_{ij} c^\dagger_i c_j$$

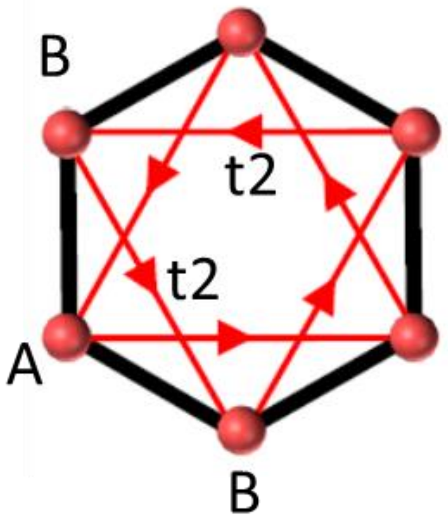
different site

loop current by Haldane

Haldane's loop current

Phys. Rev. Lett. **61** 2015 (1988)

→ Novel Prize in 2016



$$H_{\rm Haldane} = -t_1 \sum_{<ij>} c_i^\dagger c_j \; -t_2 \sum_{<<i,j>>} e^{i\phi_{ij}} c_i^\dagger c_j + e^{-i\phi_{ij}} c_j^\dagger c_i$$

effective Aharonov-Bohm (AB) phase → Origin of this imaginary hopping was unknown.

Hermite system

pure imaginary hopping
= effective AB phase

* time reversal symmetry is broken.

* odd parity

cf. AB phase in magnetic-field

$$t_{ij} = te^{i\phi_{ij}} \approx t + it\phi_{ij}$$

space inversion→ even odd

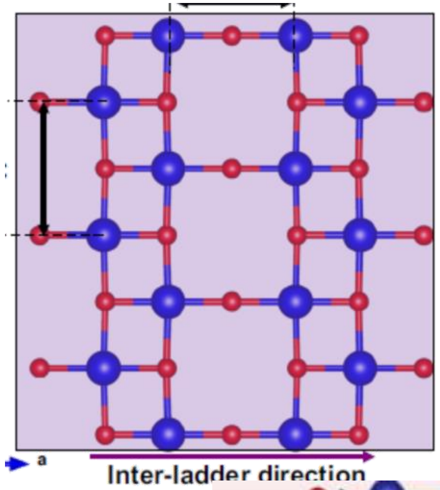
$$\phi_{ij} = -\phi_{ji}$$

$$\phi_{ij} = -e \int_{r_j}^{r_i} \vec{A} \cdot d\vec{r}$$

A: vector potential

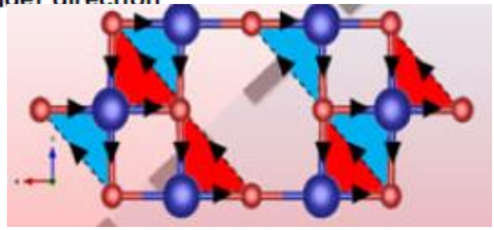
recent discover: various loop current

Cuprates experiment



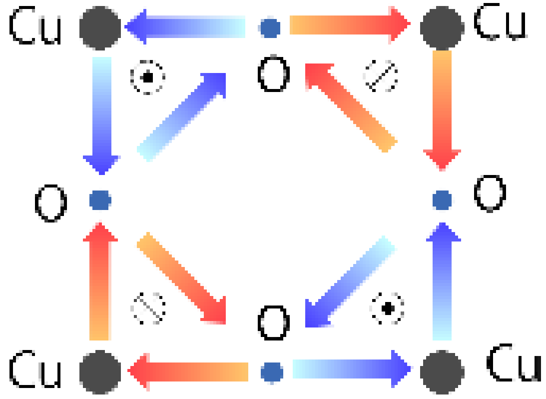
2-leg ladder

$Sr_{14}Cu_{24}O_{41}$



D Bounoua et al.,
Comm Phys 3, 123 (2020).

Varma's loop current

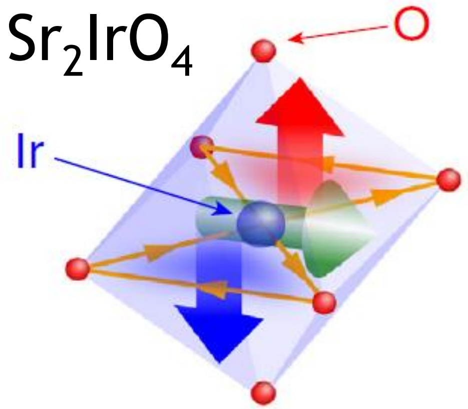


C. M. Varma,
PRB 55, 14554 (1997).
PRB 99, 224516 (2019).

I. Affleck et al.,
PRB 37, 3774 (1988).

experiment

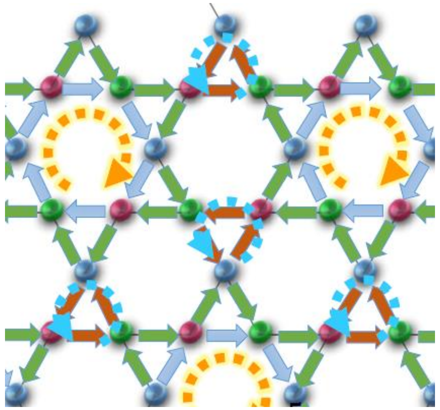
Iridate



H. Murayama *et al.*,
Phys. Rev. X **11**, 011021 (2021).

experiment

kagome



C. Mielke et al., arXiv:2106.13443.
C. Guo et al., arXiv:2203.09593

open problem

Emergence of loop currents could not be explained by RPA/ mean-field theory.

→ Can we explain by fRG? = our motivation

classification of non-local order

non-local order

$$\hat{O} = \sum_{i \neq j} \delta t_{ij}^{\sigma} c_{i\sigma}^{\dagger} c_{j\sigma}$$

Fourier transformation →

$$\hat{O} = \sum_{k,q} f_q^{\sigma}(k)\, c_{k+q,\sigma}^{\dagger} c_{k\sigma}$$

We call $f_q^{\sigma}(k)$ "form-factor".

⇕ field theory

$f_q^{\sigma}(k)$ = symmetry breaking of self energy

$$\delta\Sigma_{k\sigma}^{q} = \phi \cdot f_{\boldsymbol{q}}^{\sigma}(\boldsymbol{k})$$

$f_q^{\sigma}(k)$

spin $\uparrow \leftrightarrow \downarrow$

- S=+1: spin-symmetric $f_q^{\uparrow}(k) = f_q^{\downarrow}(k)$
  - $i \leftrightarrow j$, $P_{bond} = +1$: bond order (TRS: T=+1)
  - $P_{bond} = -1$: charge loop current (T=-1)
- S=-1: spin-anti-symmetric $f_q^{\uparrow}(k) = -f_q^{\downarrow}(k)$
  - $P_{bond} = +1$: spin-bond order (T=-1)
  - $P_{bond} = -1$: spin loop current (T=+1)

$S * P_{bond} * T = 1$

Various non-local order are explained by "non-local $f_q(k)$".

RT et al., arXiv:2205.02280 (2022)
to be published in Phys. Rev. B

study of non-local order by fRG

fRG + “optimized non-local form factor”

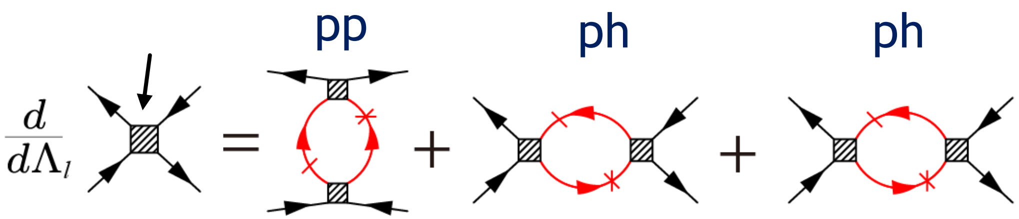
effective interaction (with vertex correction)

↓

3-point vertex

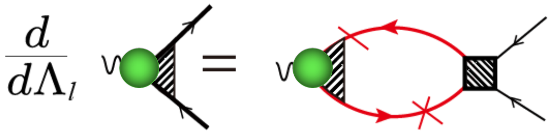
↓

Susceptibility



$\Lambda_l = W_b e^{-l}$ on-shell low energy

energy cutoff



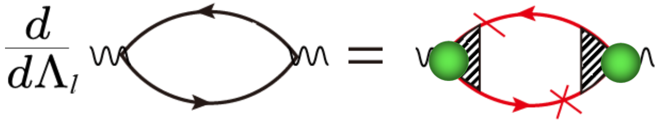
● = form-factor fq(k)

We optimize the form-factor fq(k).

$$f_k^q = \sum_{n,m=1}^{7} 2a_{nm}^q h_n(k_x) h_m(k_y),$$

a=coefficient

$$h(k) = \{1, \cos k, \cos 2k, \cos 3k, \sin k, \sin 2k, \sin 3k\}$$



higher-order many-body effects are considered.

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⇩

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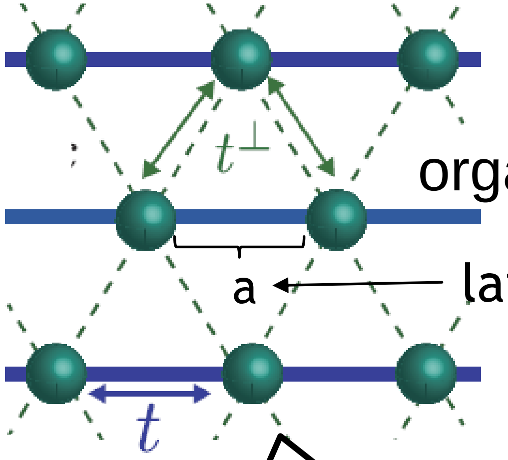
RT *et al.*, Sci. Adv. **8**, eabl4108 (2022).

3. summary

coupled chain Hubbard model



organic metals etc.,

a ← lattice constant

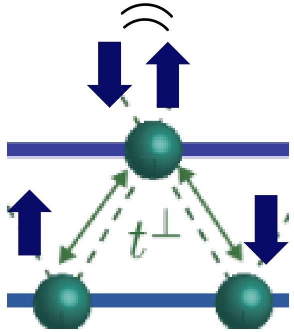
kinetic | interaction

$$\hat{H} = \sum_{ij} t_{ij} c_i^\dagger c_j + \sum_{i}\sum_{\sigma} \frac{U}{2} c_{i\sigma}^\dagger c_{i,\sigma} c_{i,\bar{\sigma}}^\dagger c_{i,\bar{\sigma}}$$

I,j: site-index

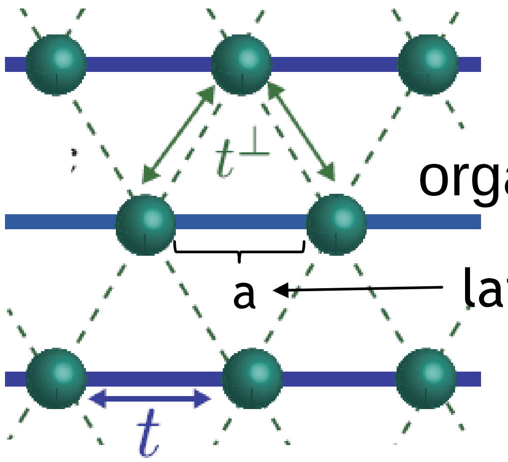
t : intra-chain hopping	$t = 1.$ fix
$t^{\perp}$: inter-chain hopping	$t^{\perp} = 0 - 0.3$
U : Coulomb repulsion	$U = 2$ fix

spin frustration



→Spin order is suppressed.

coupled chain Hubbard model



organic metals etc.,

lattice constant

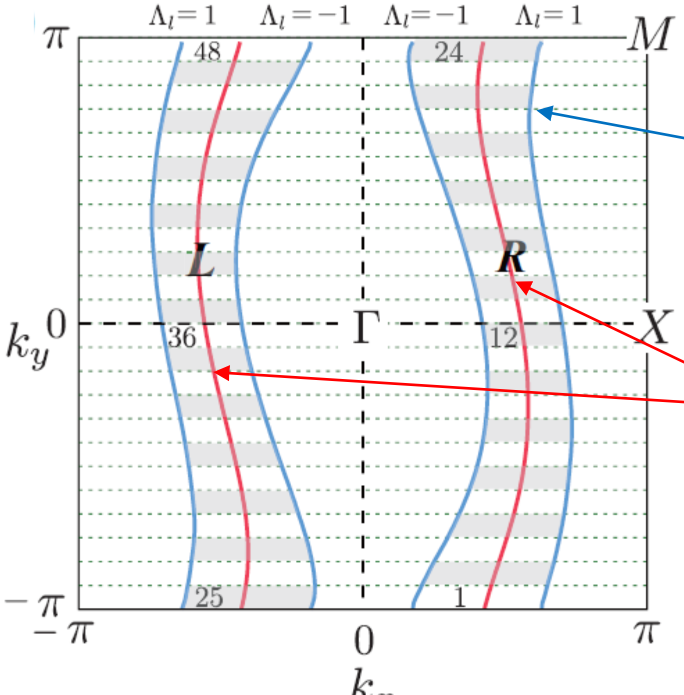
kinetic   interaction

$$\hat{H} = \sum_{ij} t_{ij} c_i^{\dagger} c_j + \sum_{i\ \sigma} \frac{U}{2} c_{i\sigma}^{\dagger} c_{i,\sigma} c_{i,\bar{\sigma}}^{\dagger} c_{i,\bar{\sigma}}$$

I,j: site-index

t : intra-chain hopping	$t = 1$. fix
$t^{\perp}$: inter-chain hopping	$t^{\perp} = 0 - 0.3$
U : Coulomb repulsion	$U = 2$ fix

48-patches in k-space



Brillouin zone

↓

absence of UV divergence

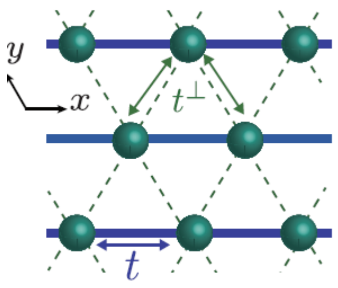
blue: constant energy line

2 Fermi surface

$$\epsilon_k = \epsilon_F$$

coupled chain Hubbard model

RT et al., Phys. Rev. B 103, L161112 (2021)

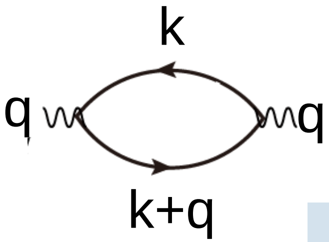


$$\hat{H} = \sum_{ij} t_{ij} c_i^{\dagger} c_j + \sum_{i\ \sigma} \frac{U}{2} c_{i\sigma}^{\dagger} c_{i,\sigma} c_{i,\bar{\sigma}}^{\dagger} c_{i,\bar{\sigma}}$$ → SU(2)-symmetry

kinetic interaction

We calculate <u>generalized susceptibility</u> by fRG.

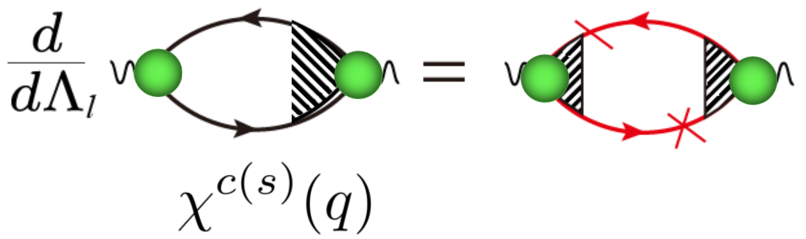
with non-local form factor



generalized susceptibility

$$\chi^{c(s)}(q) = \int_0^{\beta} d\tau \frac{1}{2} \left\langle A^{c(s)}(q,\tau) A^{c(s)}(-q,0) \right\rangle e^{i\omega_n \tau}$$

→



$$A^{c(s)}(q) = \sum_{k\sigma\sigma'} \sigma^{0(z)}_{\sigma\sigma'} f_k^q c_{k+q\sigma}^{\dagger} c_{k\sigma'}$$

$$f_k^q = \sum_{n,m=1}^{7} 2a_{nm}^q h_n(k_x) h_m(k_y),$$

coefficient a is optimized.

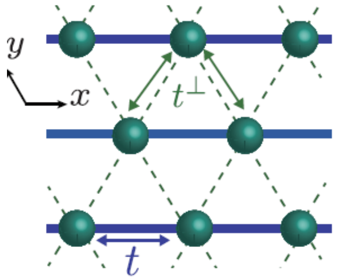
n=1 n=7

$$h(k) = \{1, \cos k, \cos 2k, \cos 3k, \sin k, \sin 2k, \sin 3k\}$$

f_k^q is optimized
to maximize $\chi^{c(s)}(q)$.

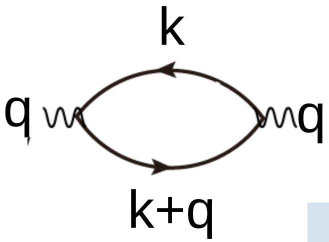
coupled chain Hubbard model

RT et al., Phys. Rev. B 103, L161112 (2021)



$$\hat{H} = \underbrace{\sum_{ij} t_{ij} c_i^{\dagger} c_j}_{\text{kinetic}} + \underbrace{\sum_{i\ \sigma} \frac{U}{2} c_{i\sigma}^{\dagger} c_{i,\sigma} c_{i,\bar{\sigma}}^{\dagger} c_{i,\bar{\sigma}}}_{\text{interaction}}$$

→ SU(2)-symmetry



We calculate generalized susceptibility by fRG.

generalized susceptibility

$$\chi^{c(s)}(q) = \int_0^{\beta} d\tau \frac{1}{2} \left\langle A^{c(s)}(q,\tau) A^{c(s)}(-q,0) \right\rangle e^{i\omega_n \tau}$$

$$A^{c(s)}(q) = \sum_{k\sigma\sigma'} \sigma^{0(z)}_{\sigma\sigma'} f_k^q c^{\dagger}_{k+q\sigma} c_{k\sigma'}$$

$$f_k^q = \sum_{n,m=1}^{7} 2a^q_{nm} h_n(k_x) h_m(k_y),$$

coefficient a is optimized.

n=1 n=7

$$h(k) = \{1, \cos k, \cos 2k, \cos 3k, \sin k, \sin 2k, \sin 3k\}$$

T=0 ε_F T=∞

quantum fluctuation thermal fluctuation

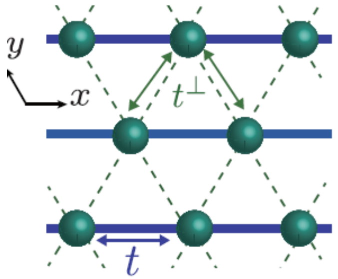
our interest = T<<ε_F

Quantum fluctuation is important !

f_k^q is optimized to maximize $\chi^{c(s)}(q)$.

coupled chain Hubbard model

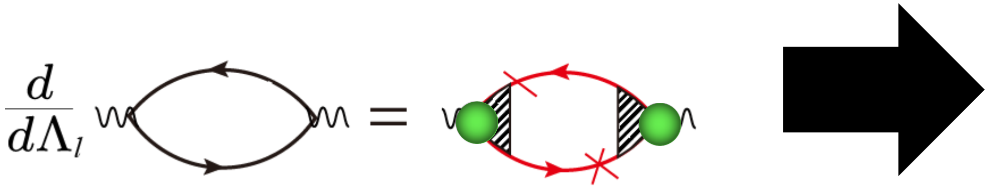
RT et al., Phys. Rev. B 103, L161112 (2021)



$$\hat{H} = \sum_{ij} t_{ij} c_i^\dagger c_j + \sum_{i\ \sigma} \frac{U}{2} c_{i\sigma}^\dagger c_{i,\sigma} c_{i,\bar{\sigma}}^\dagger c_{i,\bar{\sigma}}$$

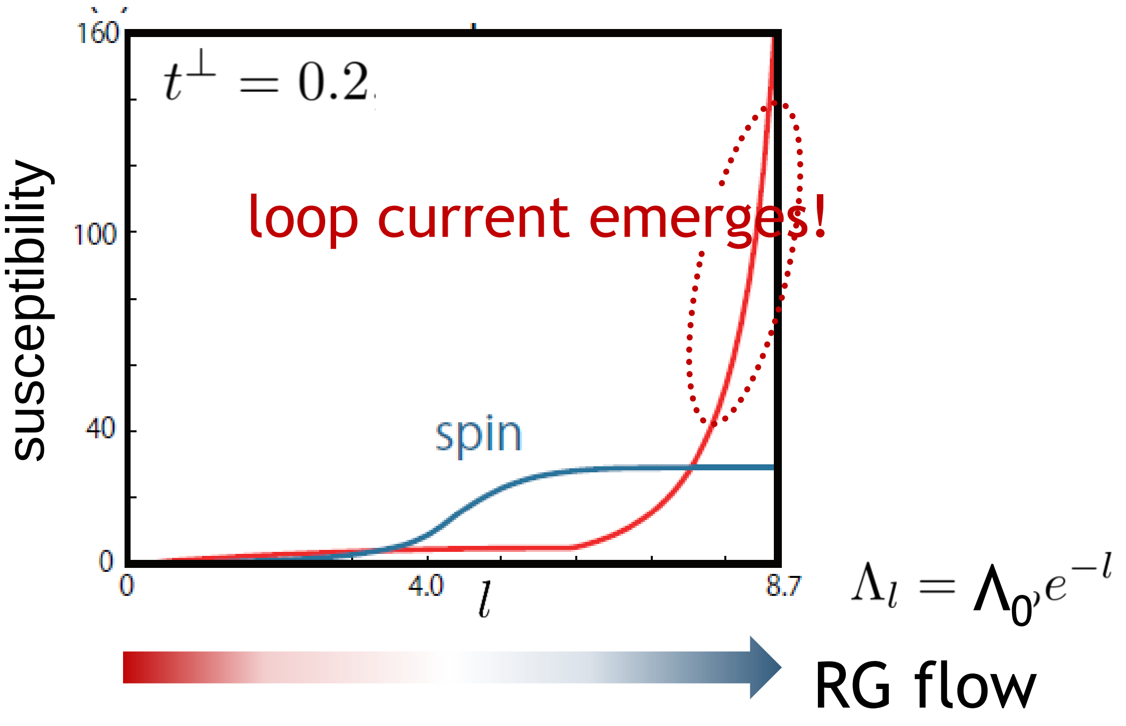
kinetic interaction

RG flow of
particle-hole susceptibility



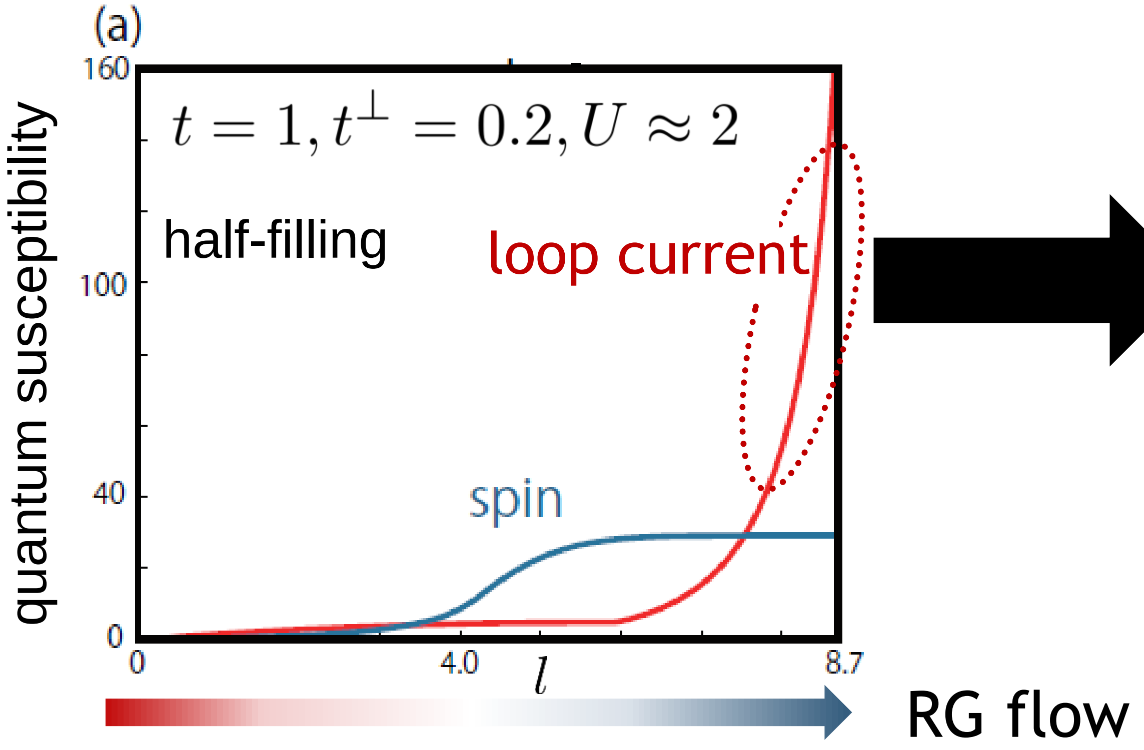
● = spin,charge, loop current, etc.,

with 49-basis

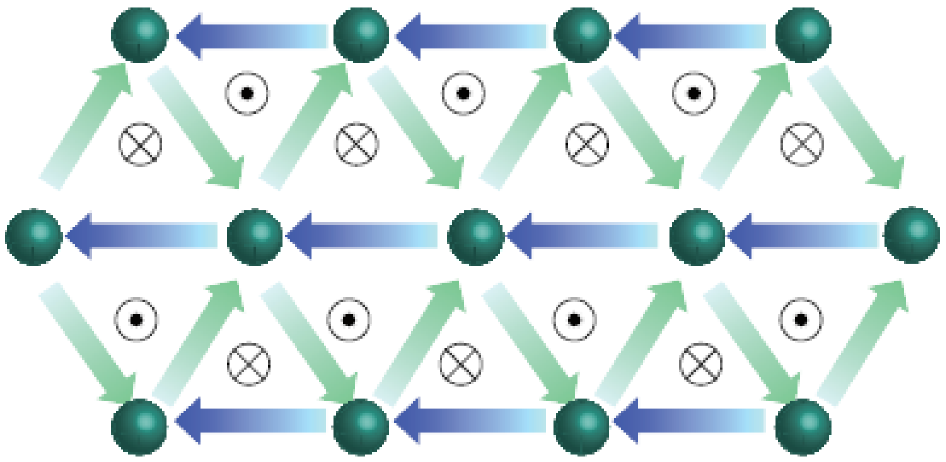


coupled chain Hubbard model

RT et al., Phys. Rev. B 103, L161112 (2021)



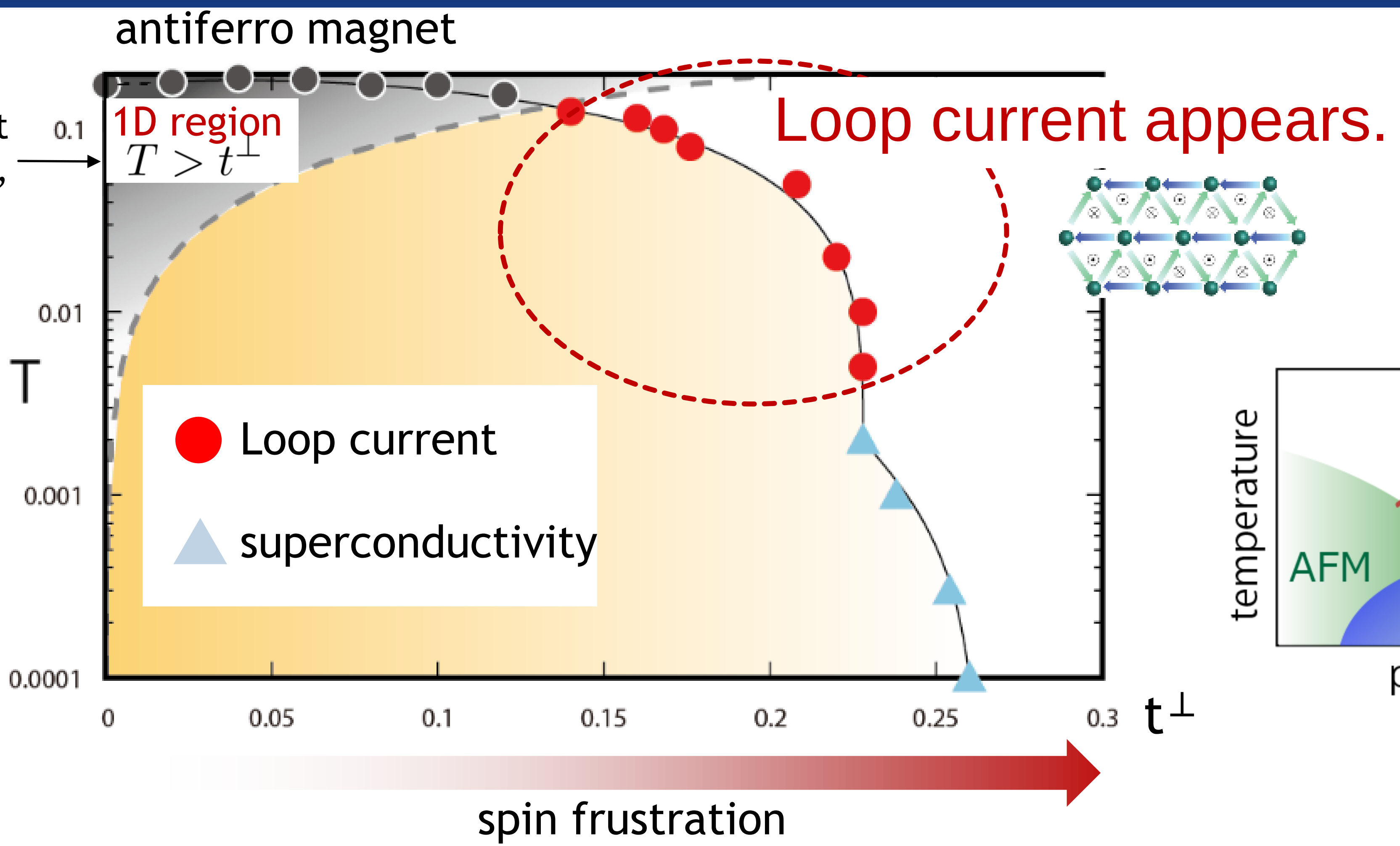
loop current emerges!

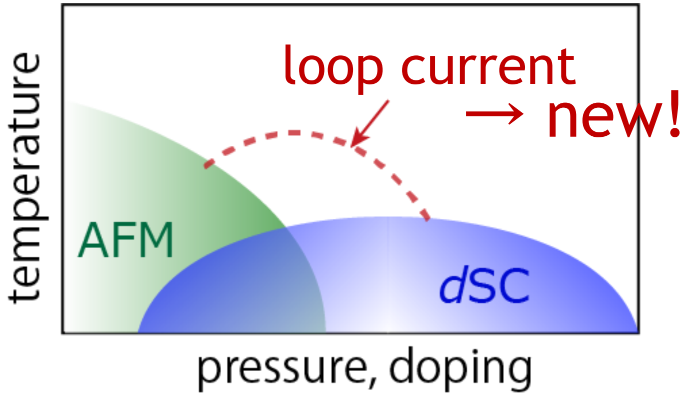


* Development of loop current fluctuation is absence in RPA

phase diagram by fRG

J. Kishine et al., JPSJ 67, 2590 (1998)





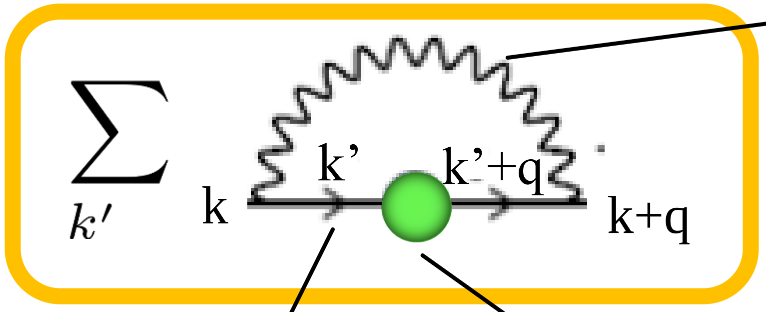
RT et al., Phys. Rev. B 103, L161112 (2021)

We discover loop current, which was overlooked for years.

origin of loop current

DW equation

$$\delta\Sigma_k^q = $$



+ mean–field

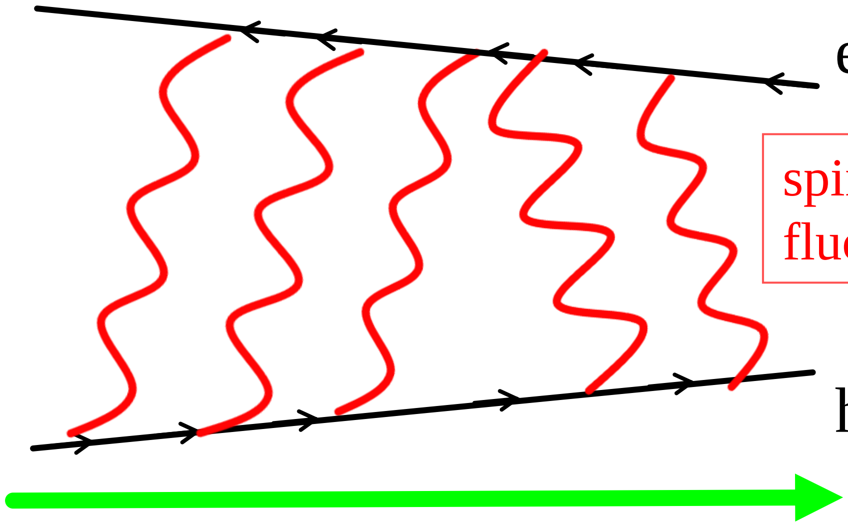
symmetry broken component of self-energy

new mechanism of loop current revealed by fRG ＆DW equation

= MT–vertex correction by spin fluctuation induce loop current.

=spin fluctuation (= local = non-local) cause loop current.

MT vertex correction



RT et al., Phys. Rev. B 103, L161112 (2021)
RT et al., Sci. Adv. 8, eabl4108 (2022).

outline

1. Introduction: fRG in condensed matter physics

2. Recent study of "non-local phase transition" based on fRG

ex. permanent loop current

①coupled-chain Hubbrad model

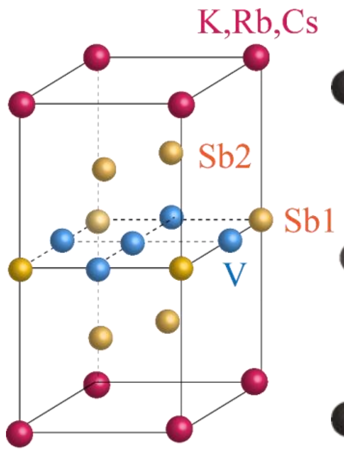
②kagome superconductor (2019~)

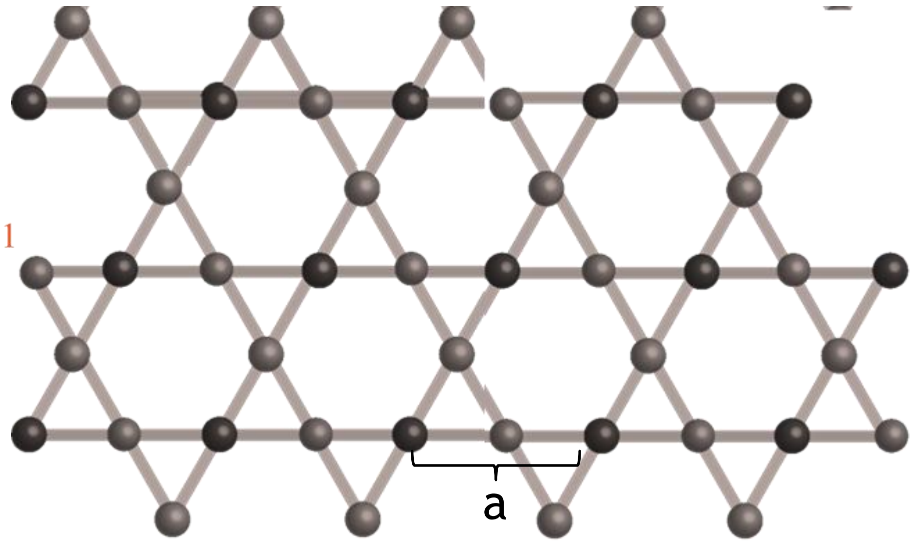
RT et al., Phys. Rev. B 103, L161112 (2021).
RT *et al.*, Sci. Adv. **8**, eabl4108 (2022).

3. summary

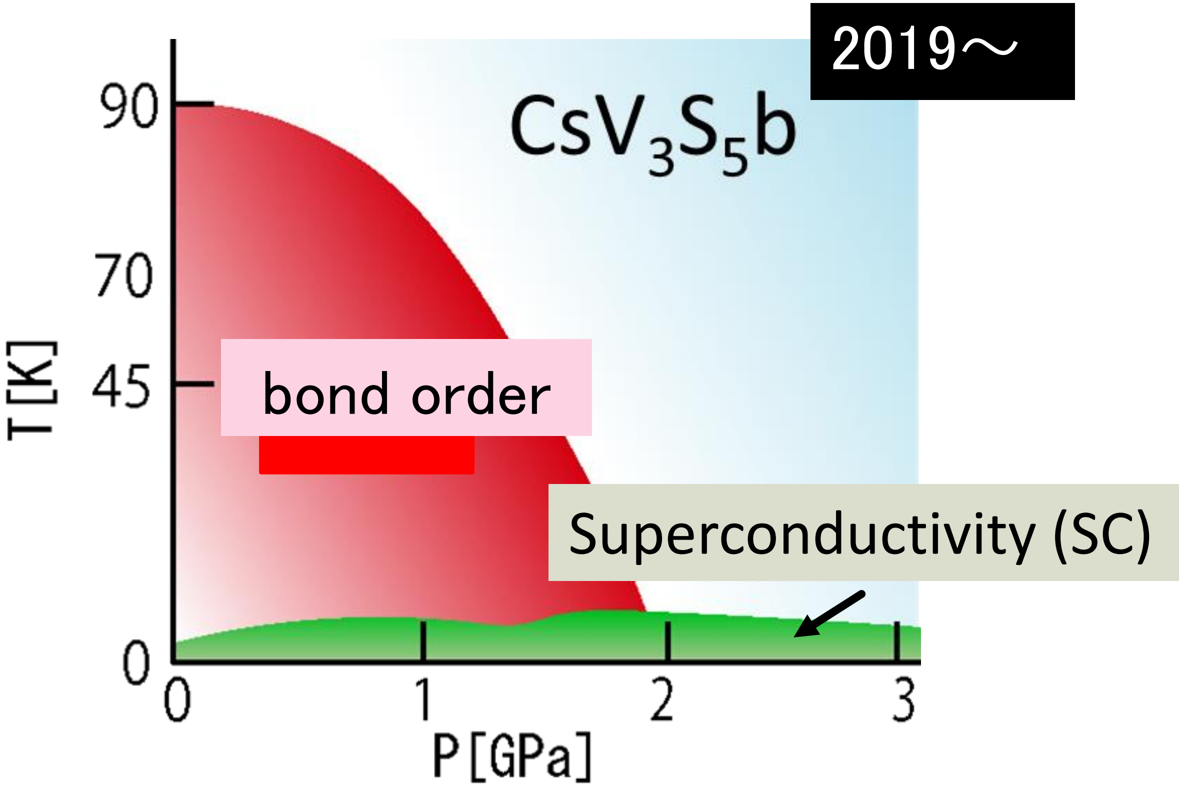
New superconductor: AV_3Sb_5 (2019~)

Kagome network of Vanadium-ion

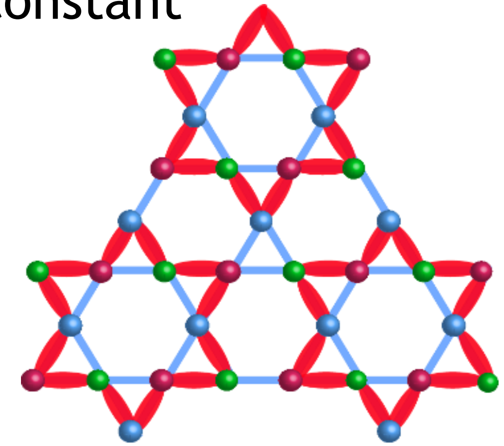




a: lattice constant



bond order (David-star)



= Hopping-integral is spontaneously enhanced.

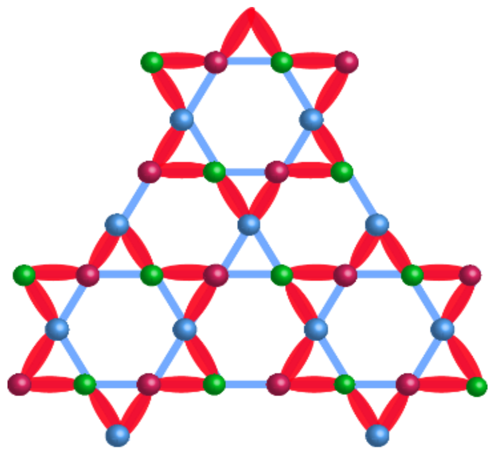
→ Translational symmetry is broken.

$$\psi(r) \neq \psi(r+a)$$

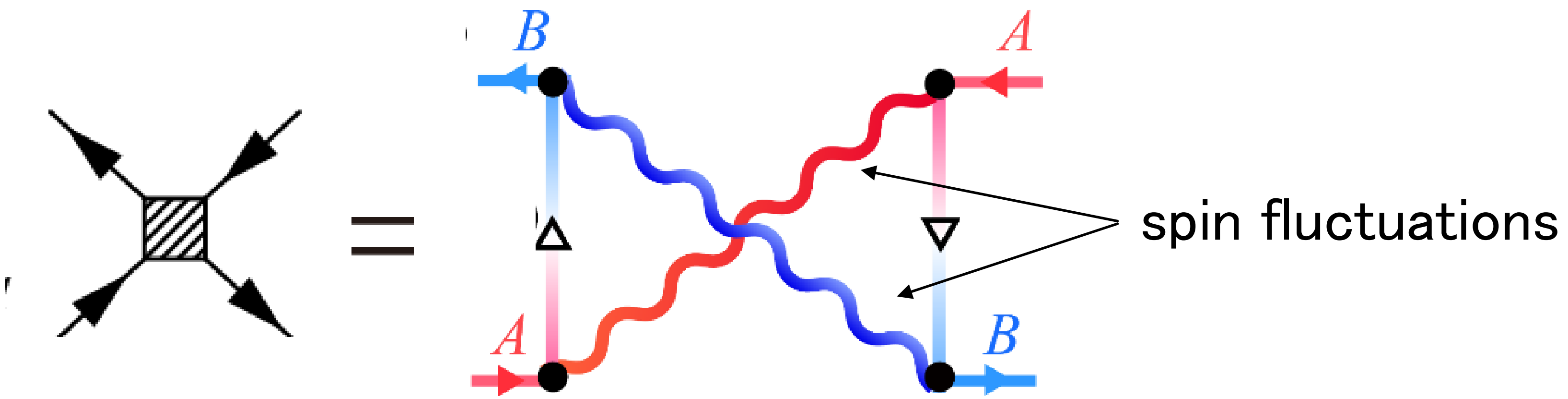
origin of bond order

R. Tazai *et al.*, Sci. Adv. **8**, eabl4108 (2022).

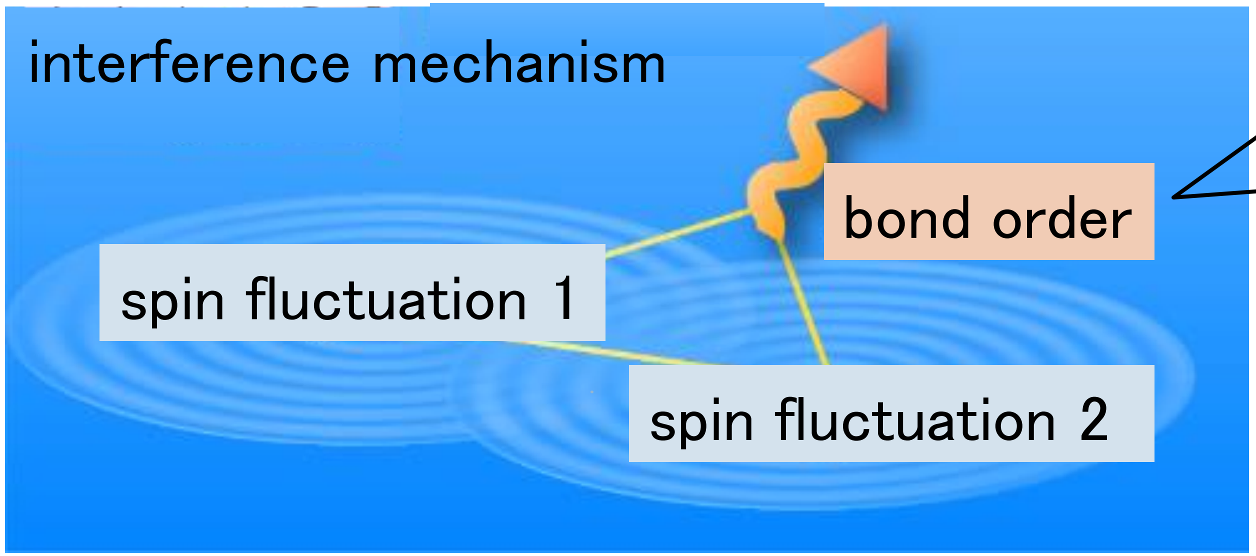
We reveal that bond order is induced by AL vertex correction.
= interference of spin-fluctuations

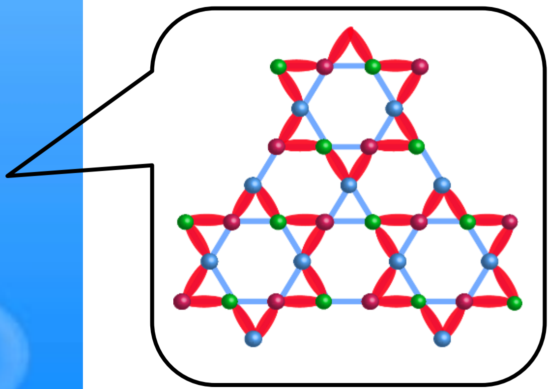


bond order (David-star)



Aslamasov-Larkin vertex correction



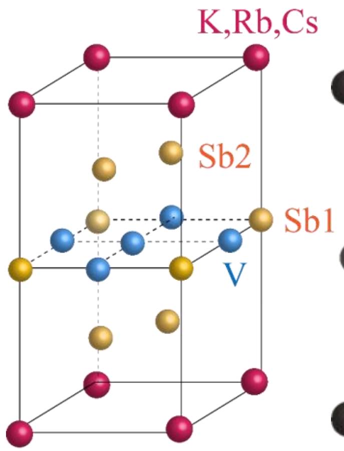


κ-(BEDT-TTF)$_2$X

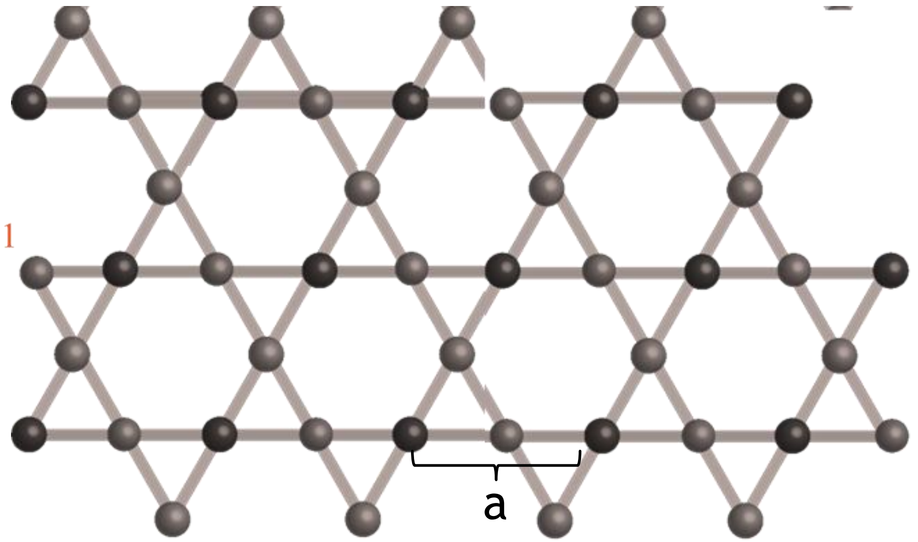
bond-order by fRG

RT et al., Phys. Rev. Research 3, 022014 (2021).

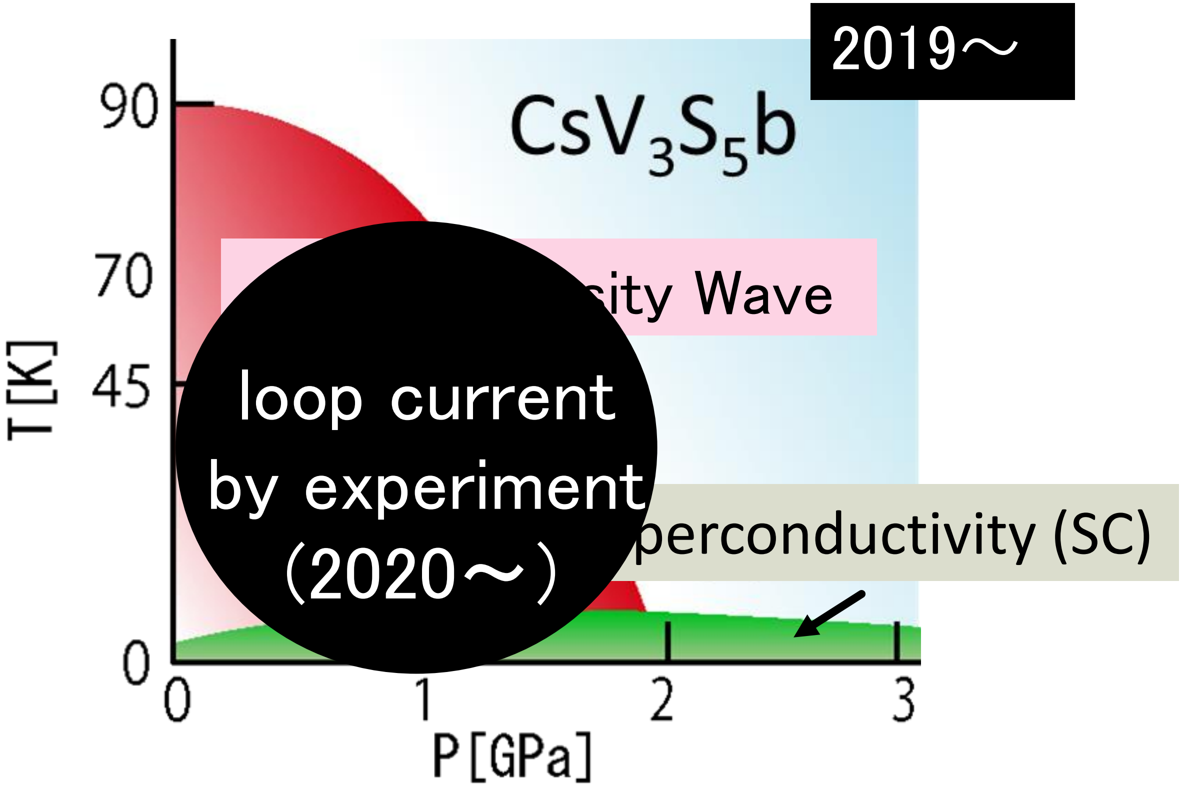
New superconductor: AV_3Sb_5 (2019~)

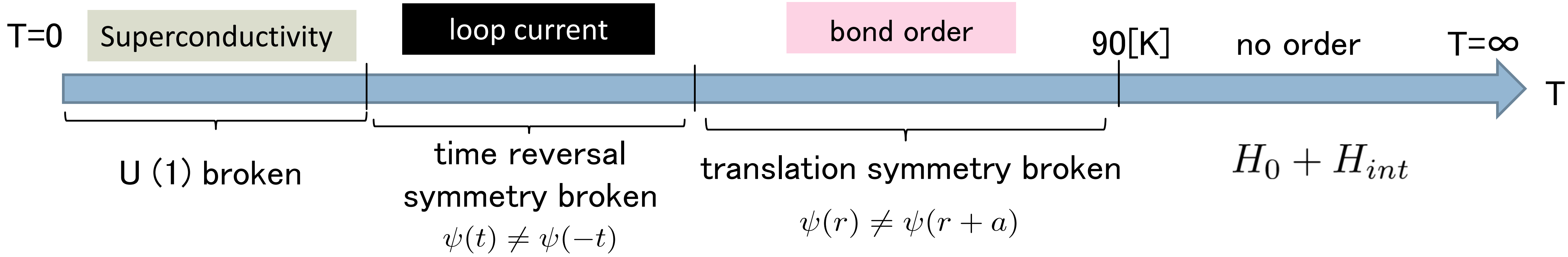


Kagome network of Vanadium-ion

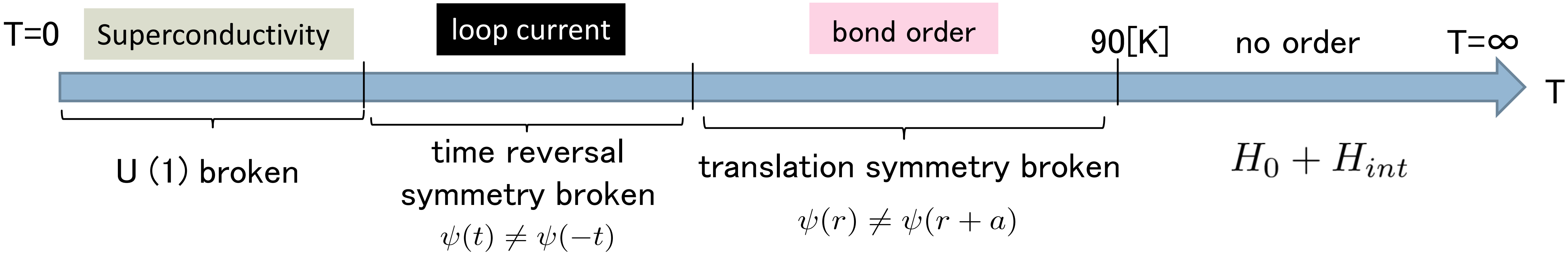


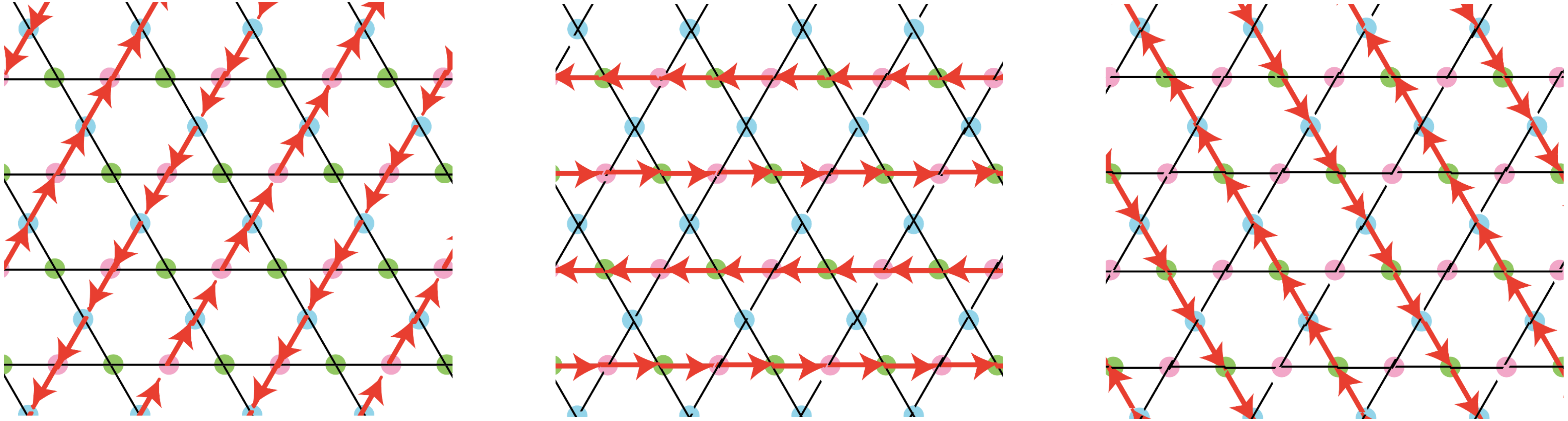
a: lattice constant





loop current in kagome metal

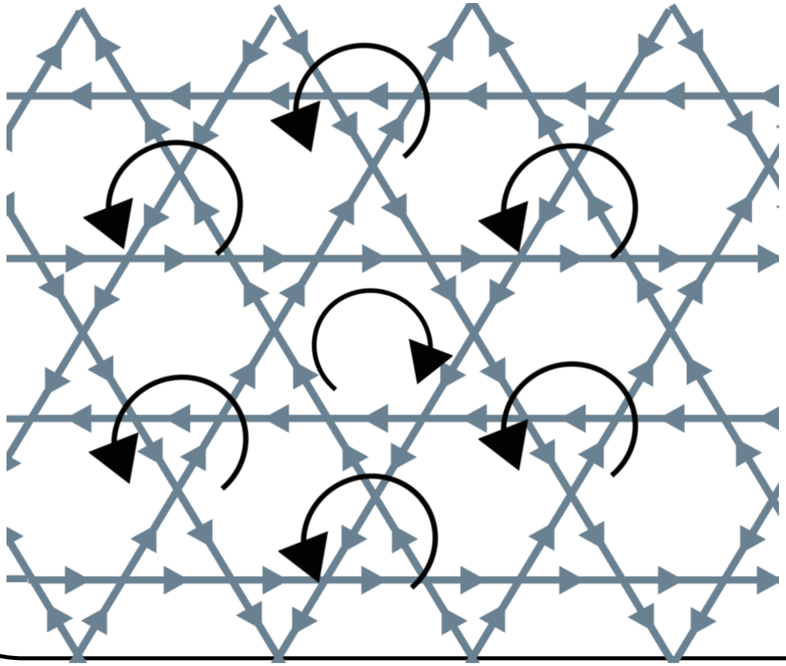




loop current in kagome metal

T=0 | Superconductivity | loop current phase | bond order | 90[K] | no order | T=∞ → T

- U (1) broken (Superconductivity)
- time reversal symmetry broken $\psi(t) \neq \psi(-t)$ (loop current phase)
- translation symmetry broken $\psi(r) \neq \psi(r+a)$ (bond order)
- $H_0 + H_{int}$ (no order)



Origin of loop current was unknown.

↓

We reveal "Maki–Thompson vertex corrections induce loop current".

R. Tazai *et al.*, arXiv:2207.08068 (2022).

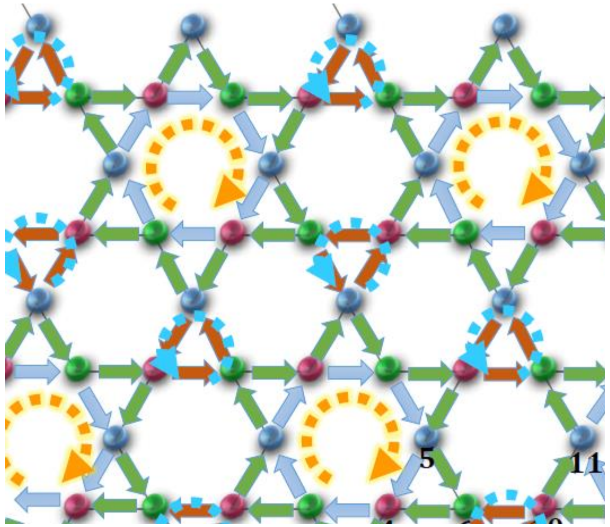
new mechanism !

loop current in kagome metal

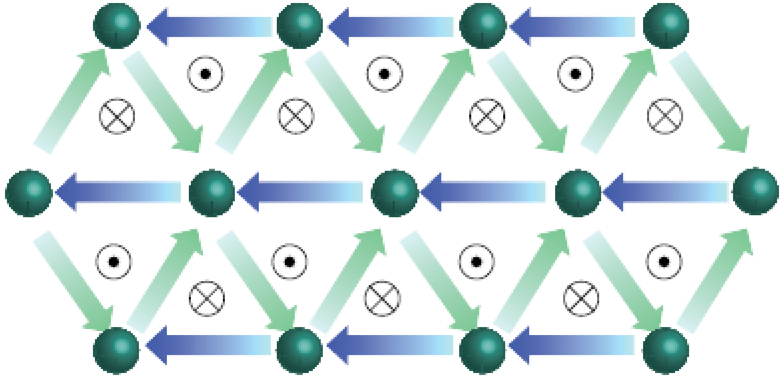
R. Tazai *et al.*, Sci. Adv. **8**, eabl4108 (2022).

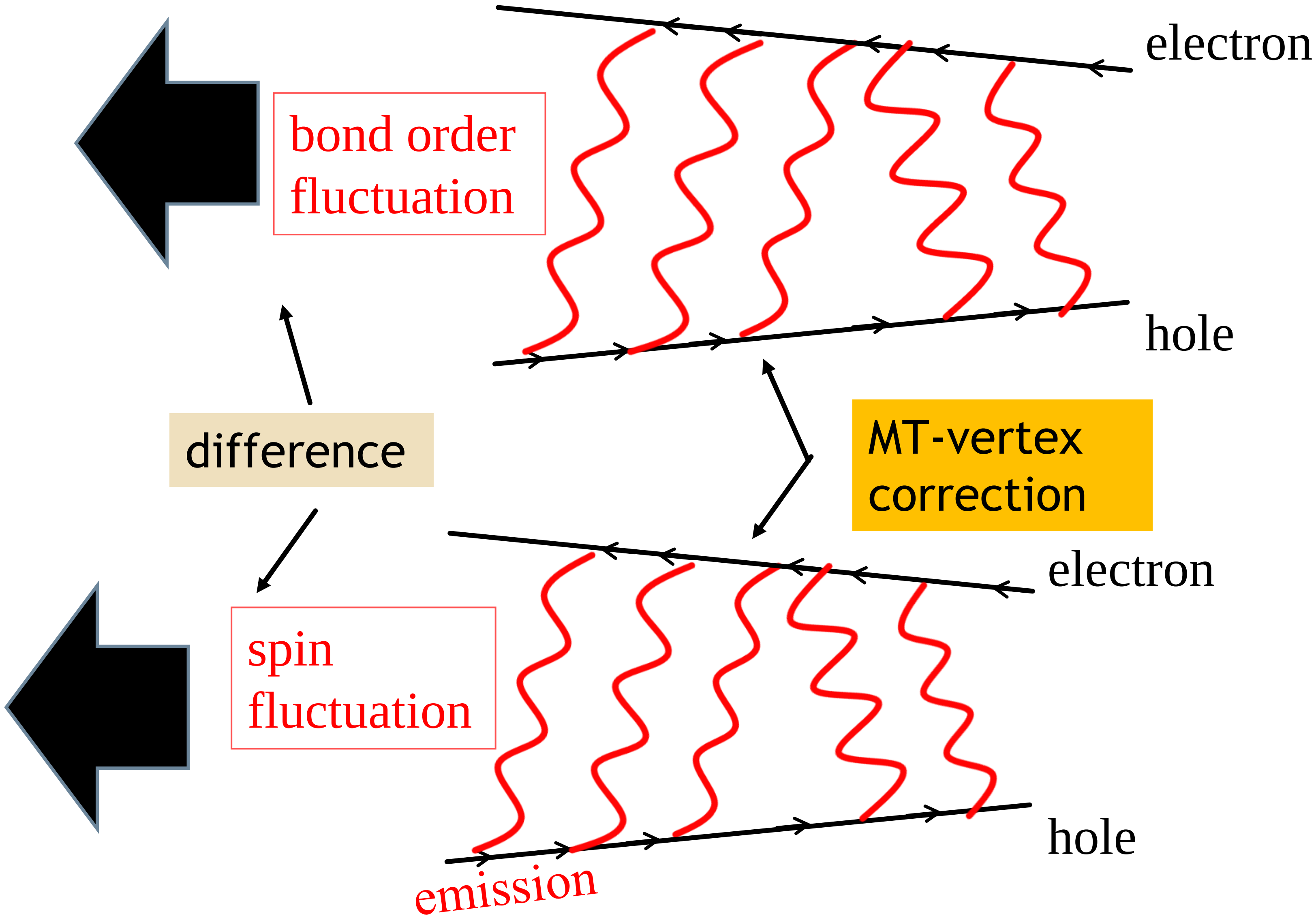
We reveal a new origin of "loop current" by Maki-Thompson vertex correction.

Kagome metal



coupled-chain model

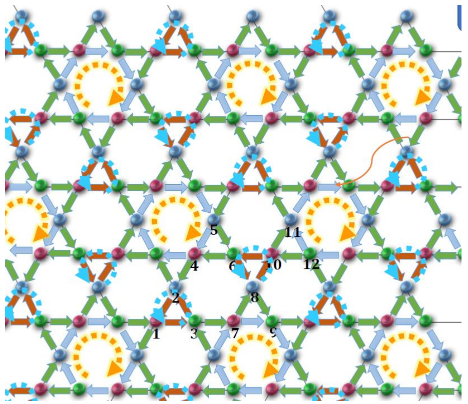




Summary

We discover the microscopic origin of new phase transition by using fRG.
ex. loop current

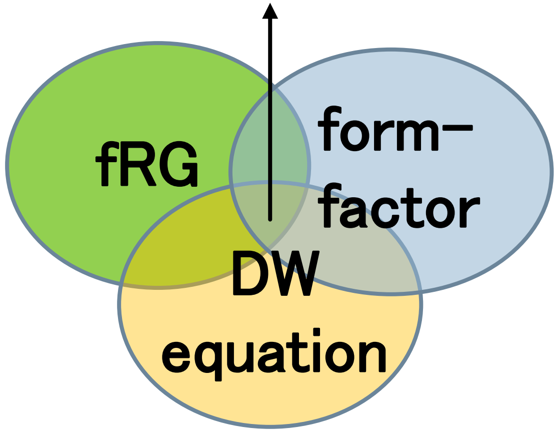
"Beyond mean-field vertex correction is necessary"



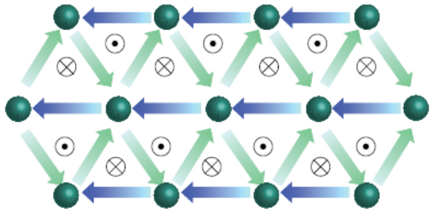
Maki-Thompson VC cause loop current in Kagome (2021～)

・static, without joule heating
space-dependent

new phase transition
(ex. loop current)



・By optimizing the form-factor in fRG, new phase transition has been revealed.



・By comparing fRG & DW equation (=diagrammatic), origin of phase transition is clearly understood.