Functional renormalization group approach to asymptotically safe gravity

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Functional Renormalization Group@RIKEN

Quantum gravity

- One says
    - Quantum gravity based on the Einstein-Hilbert action is not renormalizable.
    - Because the Newton constant is mass-dimension -2.

Quantum gravity

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  - Quantum gravity based on the Einstein-Hilbert action is not renormalizable.

    perturbatively

  - Because the Newton constant is mass-dimension -2.

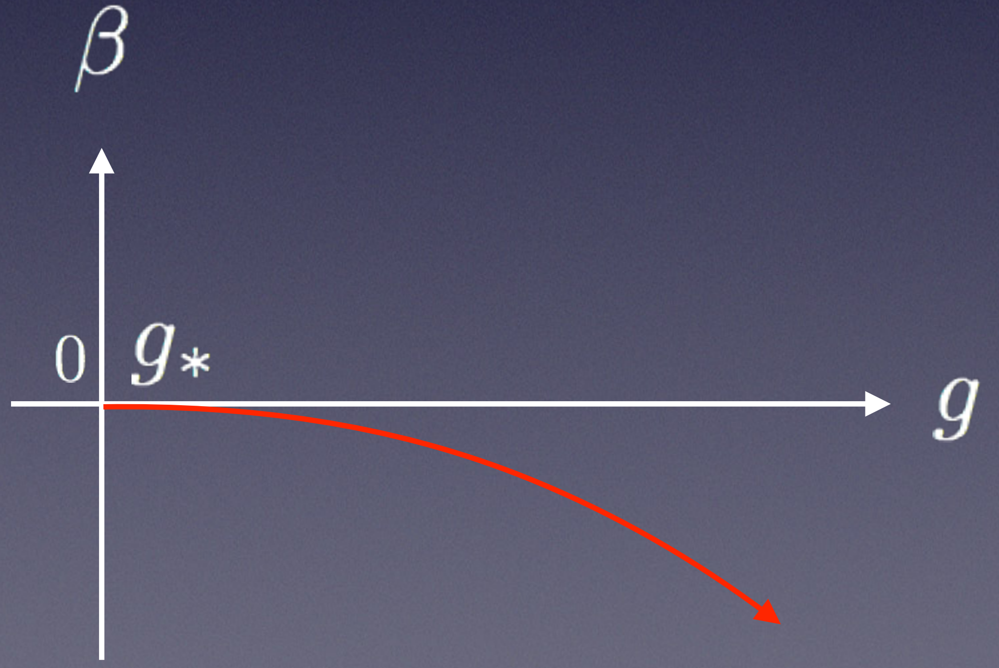
Contents

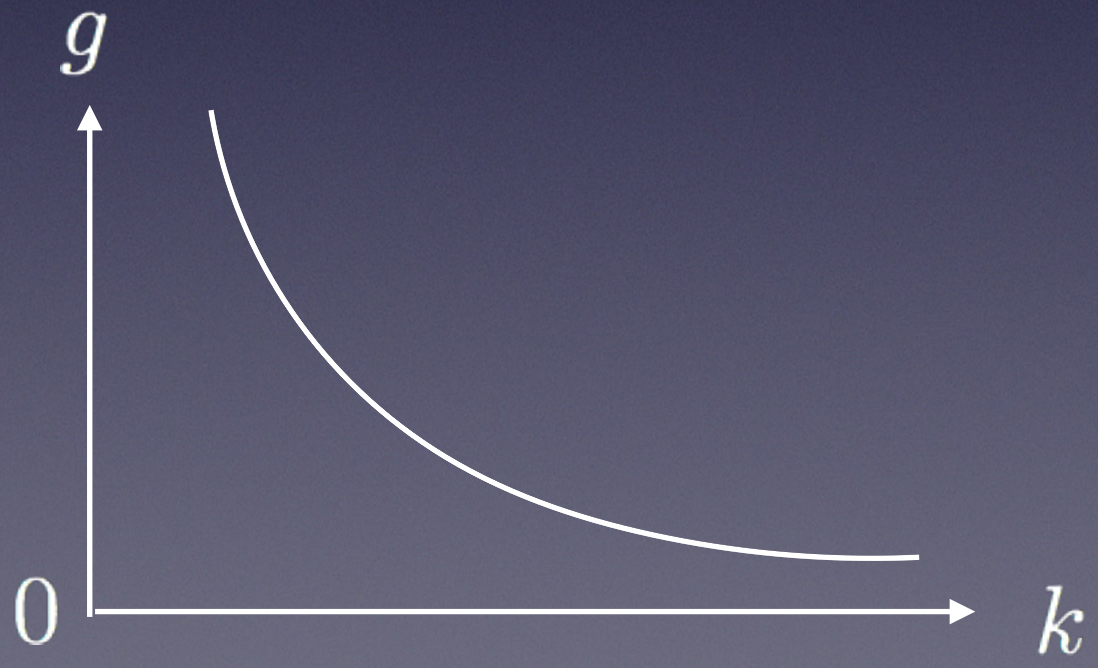
- Asymptotically safe gravity:
  - non-perturbatively renormalizable quantum gravity
- Universality class at fixed point
  - Duality between asymptotically safe and free theories
- Asymptotic safety for scalar-gravity system

Asymptotic freedom

- Asymptotic freedom

$$\partial_t g = -\beta_0 g^3, \quad \beta_0 > 0$$

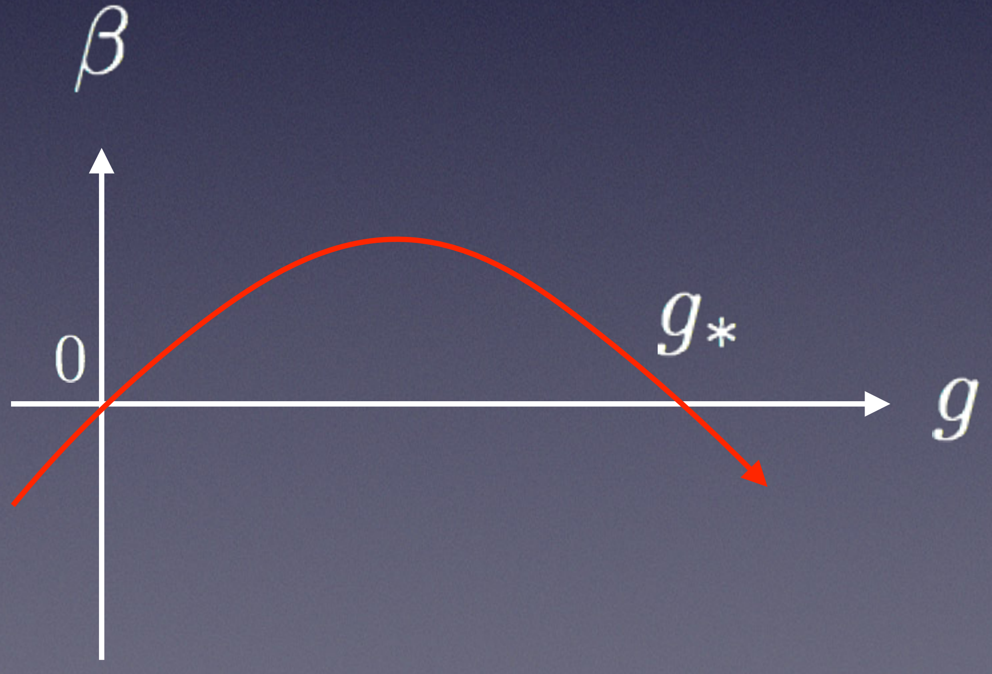


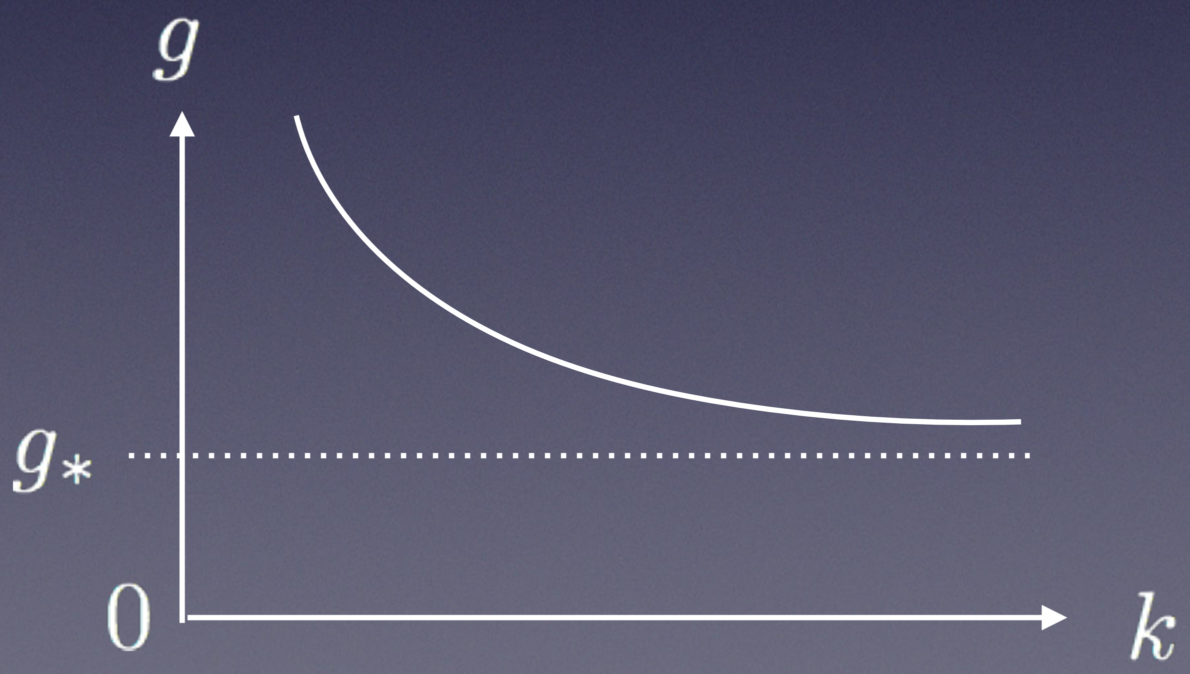


Asymptotic safety

- Asymptotic safety $g = G_N k^2$

$$\partial_t g = 2g - \beta_0 g^2, \quad \beta_0 > 0$$





What is asymptotic safety?

- In UV limit ($\Lambda \to \infty$)
  - Asymptotic freedom: asymptotically reach to a free theory (Gaussian FP). Perturbative
  - Asymptotic safety: asymptotically reach to an interacting theory (Non-trivial FP). Non-perturbative

Asymptotically safe theories

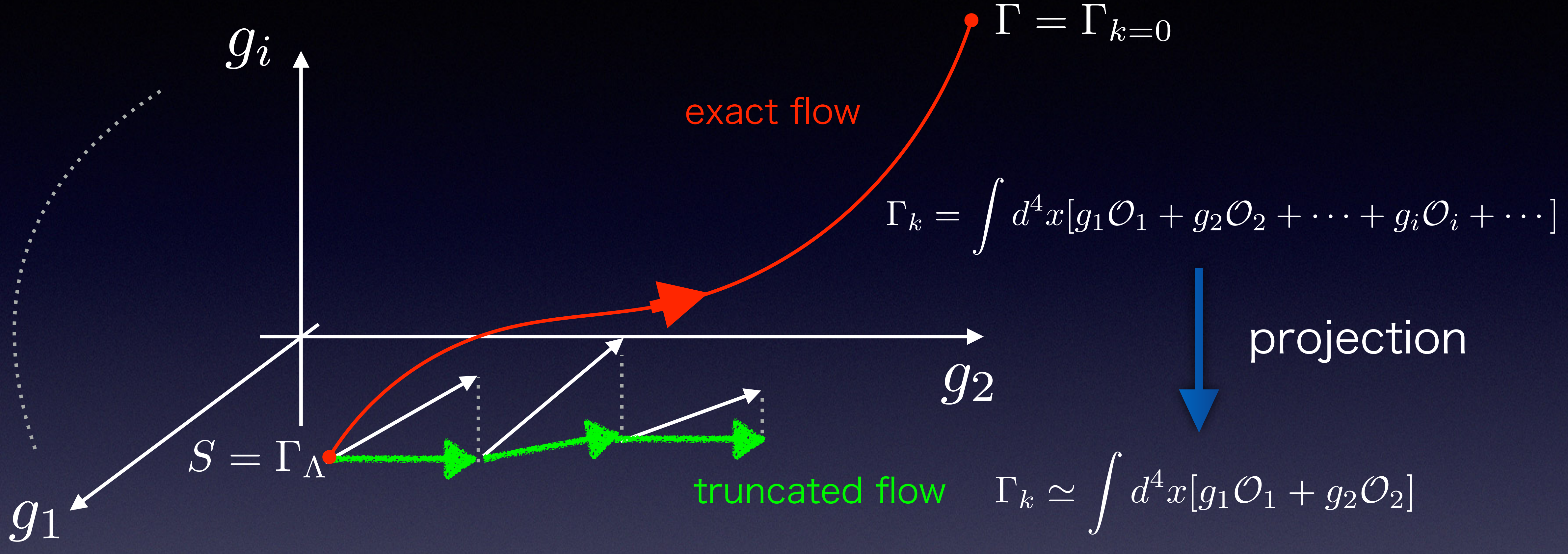
- D=3 non-linear σ model (Scalar theory)
- D=3 Gross-Neveu model (Fermionic theory)
- D=5 Yang-Mills theory???

$$\partial_t \tilde{g}^2 = \tilde{g}^2 - \frac{\tilde{g}^4}{16\pi^3}\frac{11N_c}{6} \qquad \tilde{g}^2 = kg^2$$

- D=4 gravity???

Does there exist asymptotically safe gauge theory?

Functional renormalization group



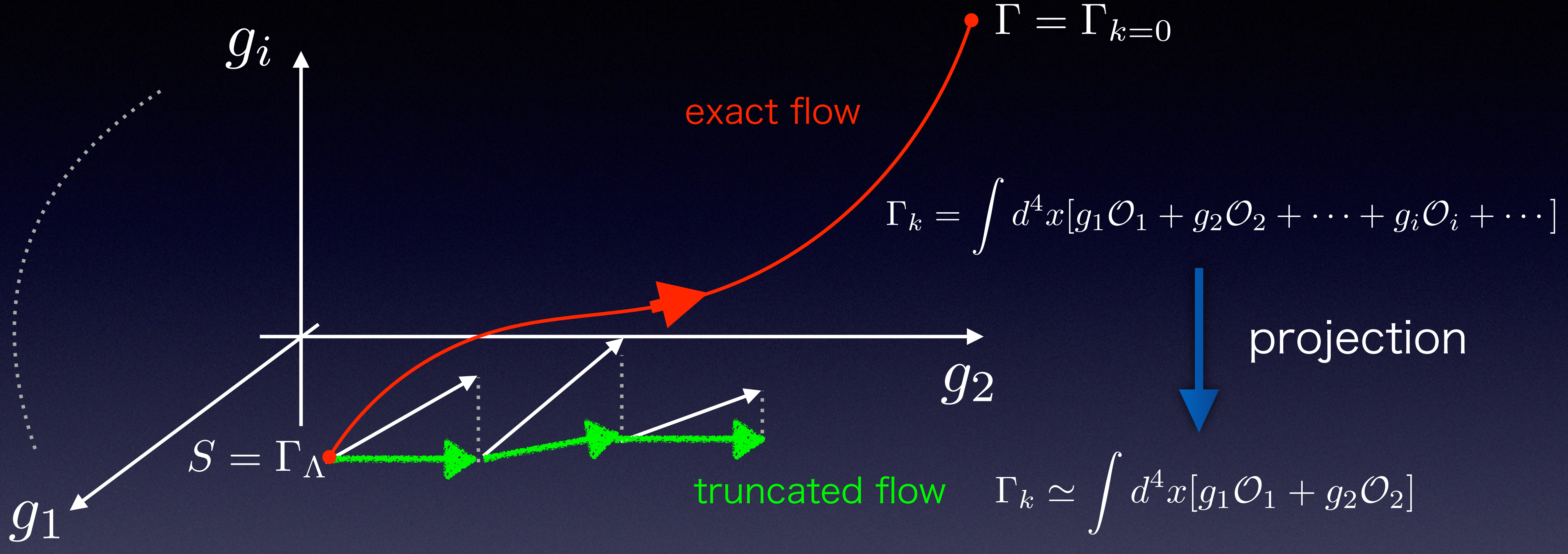
$$\Gamma_k = \int d^4x[g_1\mathcal{O}_1 + g_2\mathcal{O}_2 + \cdots + g_i\mathcal{O}_i + \cdots]$$

projection

$$\Gamma_k \simeq \int d^4x[g_1\mathcal{O}_1 + g_2\mathcal{O}_2]$$

Wetterich equation $$k\partial_k\Gamma_k = \frac{1}{2}\mathrm{Str}[(\Gamma_k^{(2)} + R_k)^{-1}k\partial_k R_k]$$

Functional renormalization group



$$\Gamma_k = \int d^4x [g_1 \mathcal{O}_1 + g_2 \mathcal{O}_2 + \cdots + g_i \mathcal{O}_i + \cdots]$$

$$\Gamma_k \simeq \int d^4x [g_1 \mathcal{O}_1 + g_2 \mathcal{O}_2]$$

Wetterich equation $$k\partial_k \Gamma_k = \frac{1}{2} \mathrm{Str}[(\Gamma_k^{(2)} + R_k)^{-1} k\partial_k R_k]$$

$$k\partial_k \Gamma_k = \int d^4x [\underbrace{(k\partial_k g_1)}_{\beta_1(g)} \mathcal{O}_1 + \underbrace{(k\partial_k g_2)}_{\beta_2(g)} \mathcal{O}_2 + \cdots + \underbrace{(k\partial_k g_i)}_{\beta_i(g)} \mathcal{O}_i + \cdots]$$

Fixed point $$k\partial_k \Gamma_k^* = 0 \longrightarrow \beta_i(g^*) = 0$$

Critical exponent

$$k\frac{dg_i}{dk} = \beta_i(g)$$

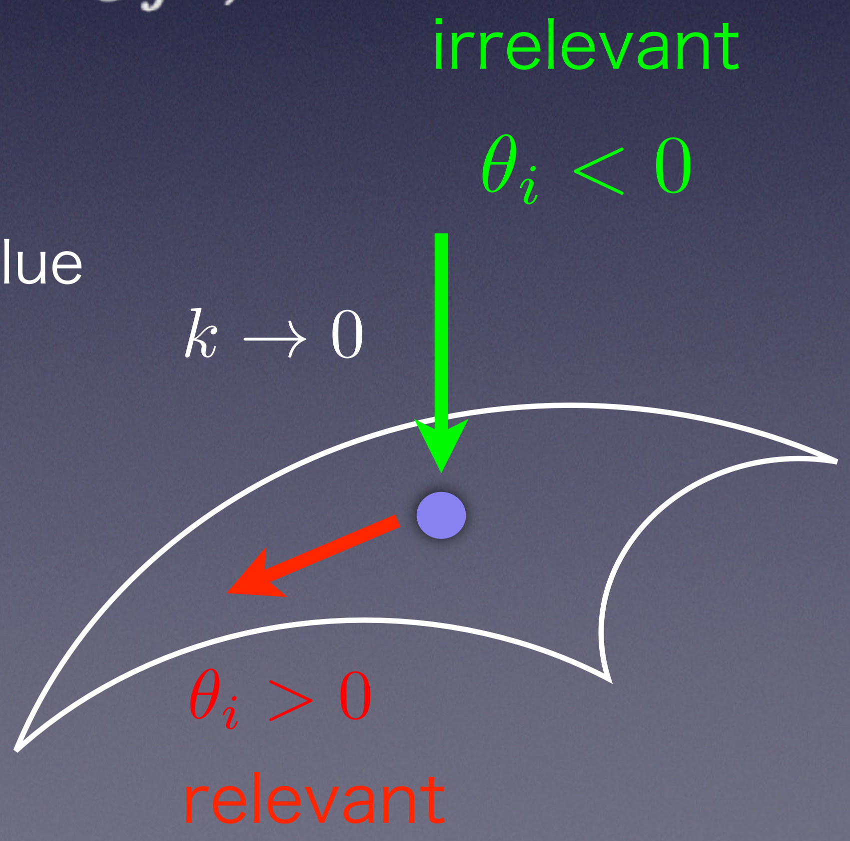
- RG eq. around FP g*

$$k\frac{dg_i}{dk} \simeq \beta_i(\cancel{g_*}) + \left.\frac{\partial \beta_i}{\partial g_j}\right|_{g=g_*}(g_j - g_{j*})$$

- Solution of RG eq.

- eigenvalue

$$g_i(k) = g_i^* + \sum_j^N \zeta_j^i \left(\frac{k}{\Lambda}\right)^{-\theta_j}$$



irrelevant

$\theta_i < 0$

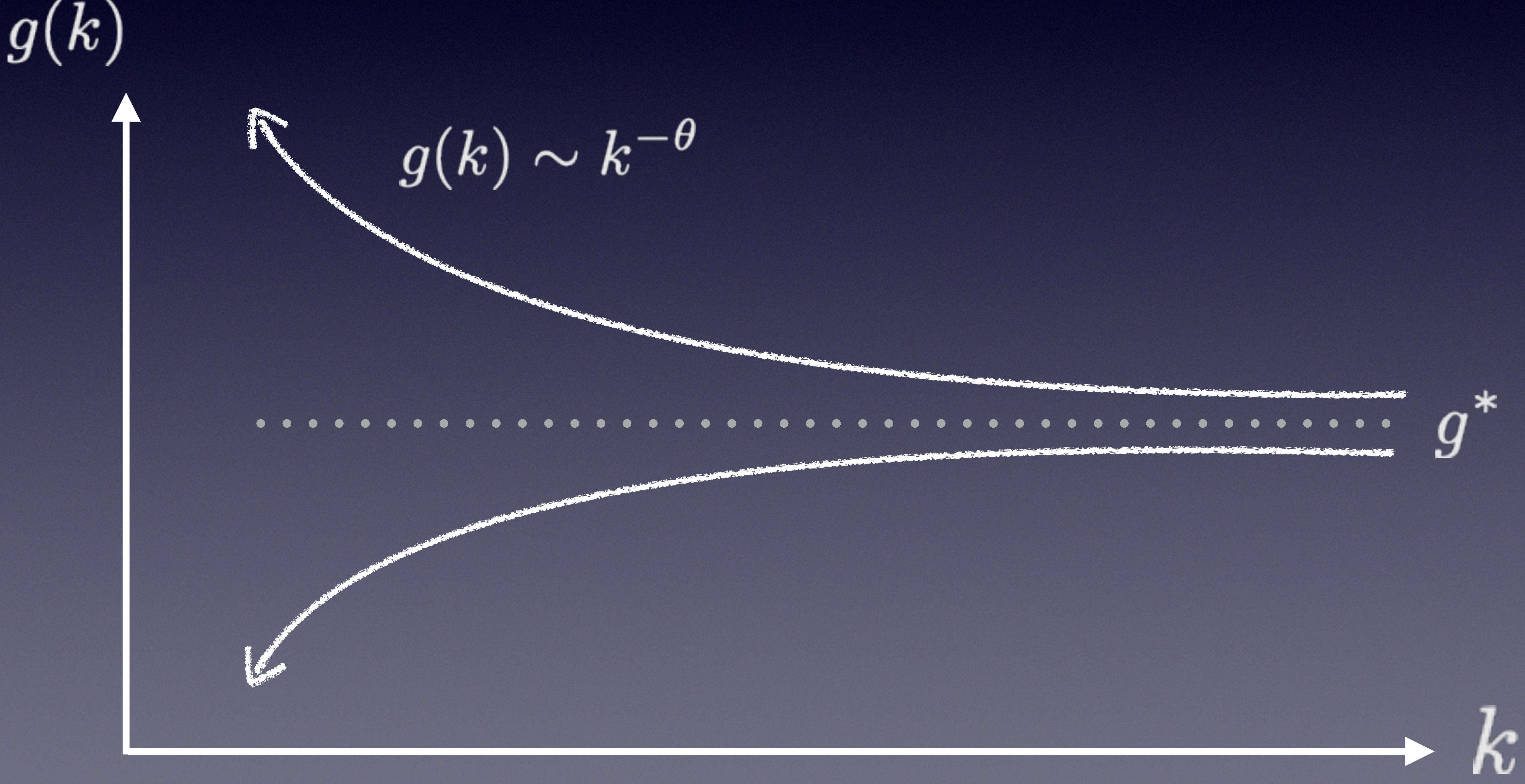
$k \to 0$

$\theta_i > 0$

relevant

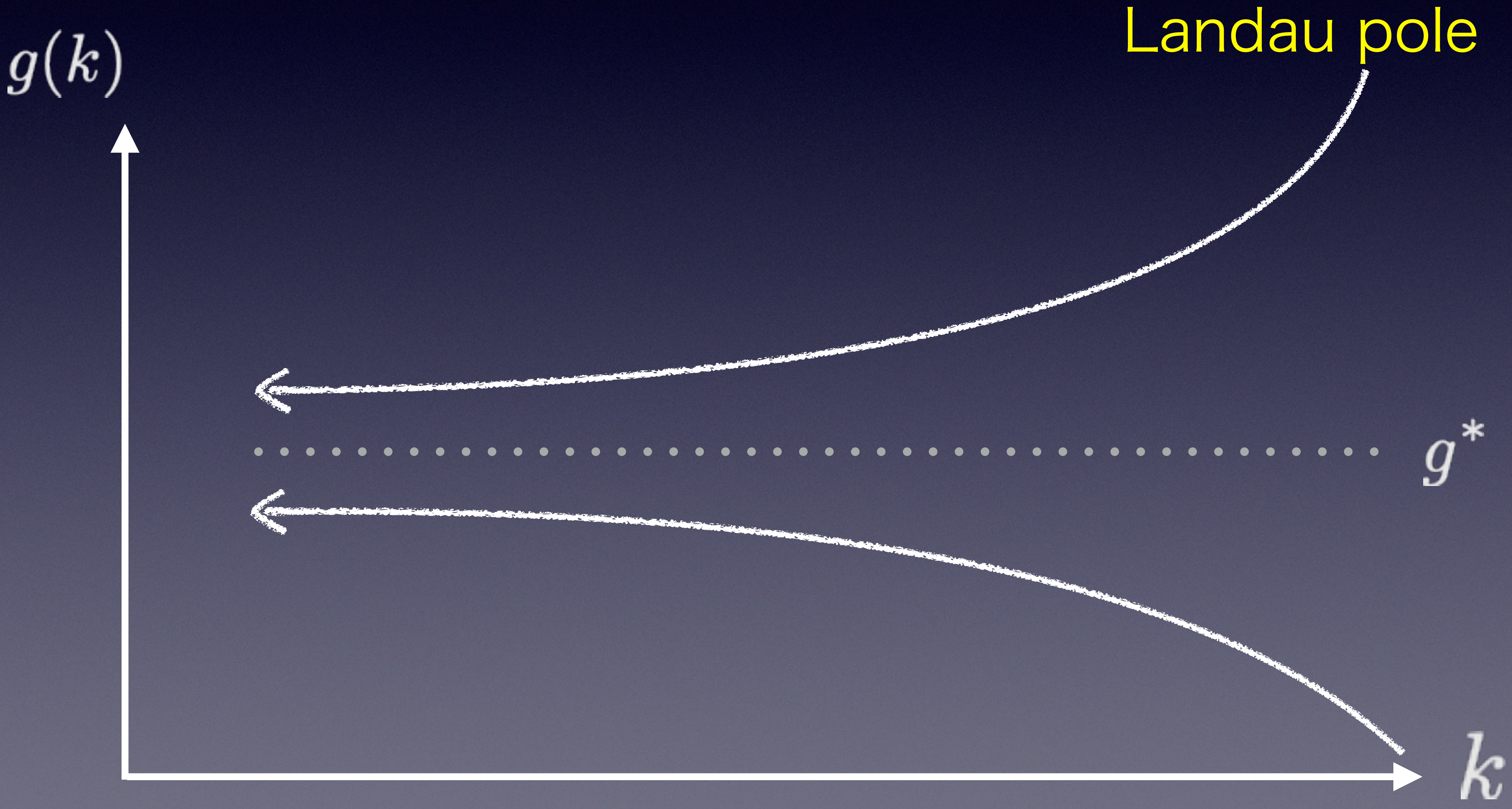
Relevant: $\theta > 0$

- Free parameter



Irrelevant $\theta < 0$

- Predictable parameter



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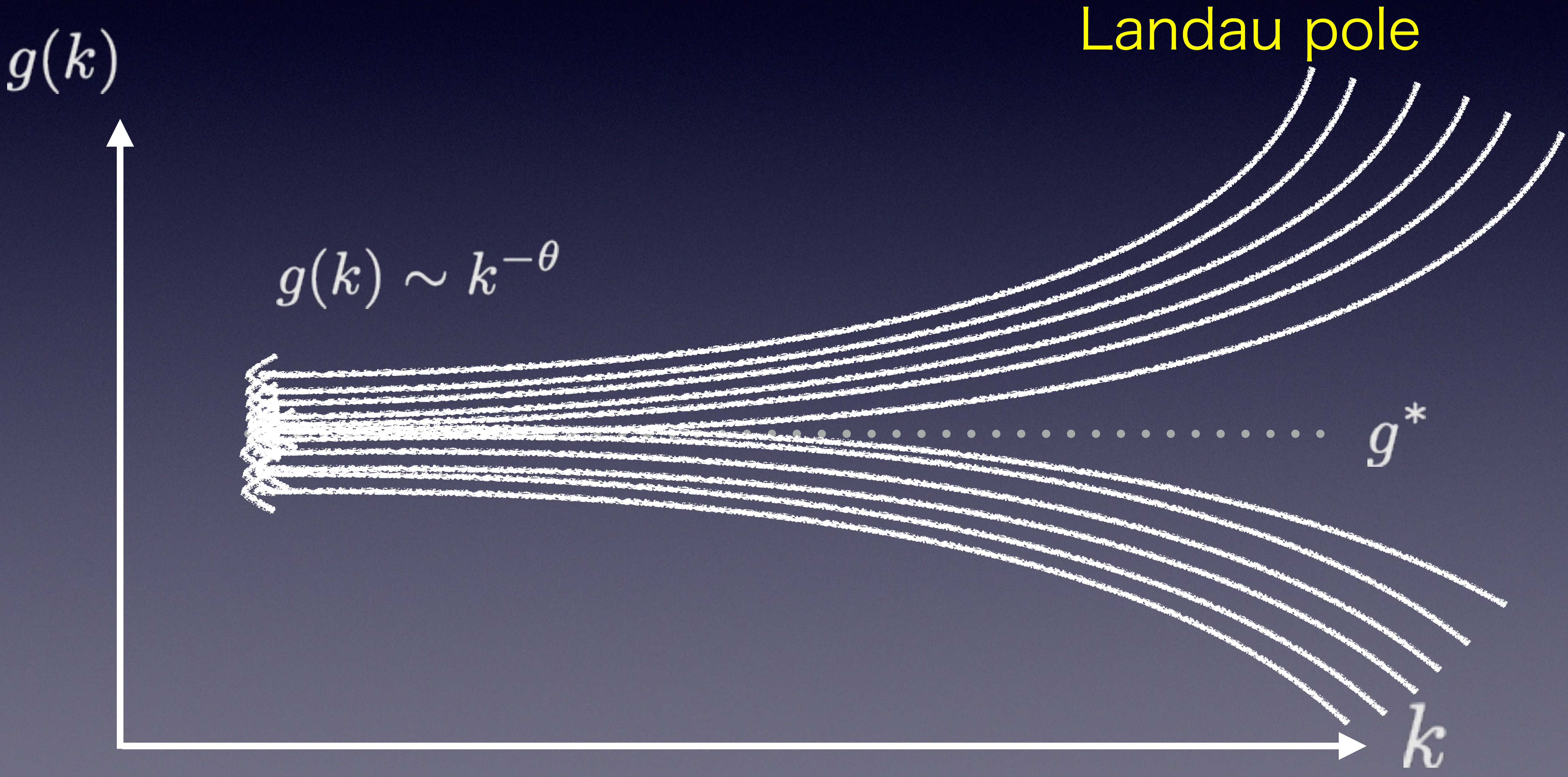
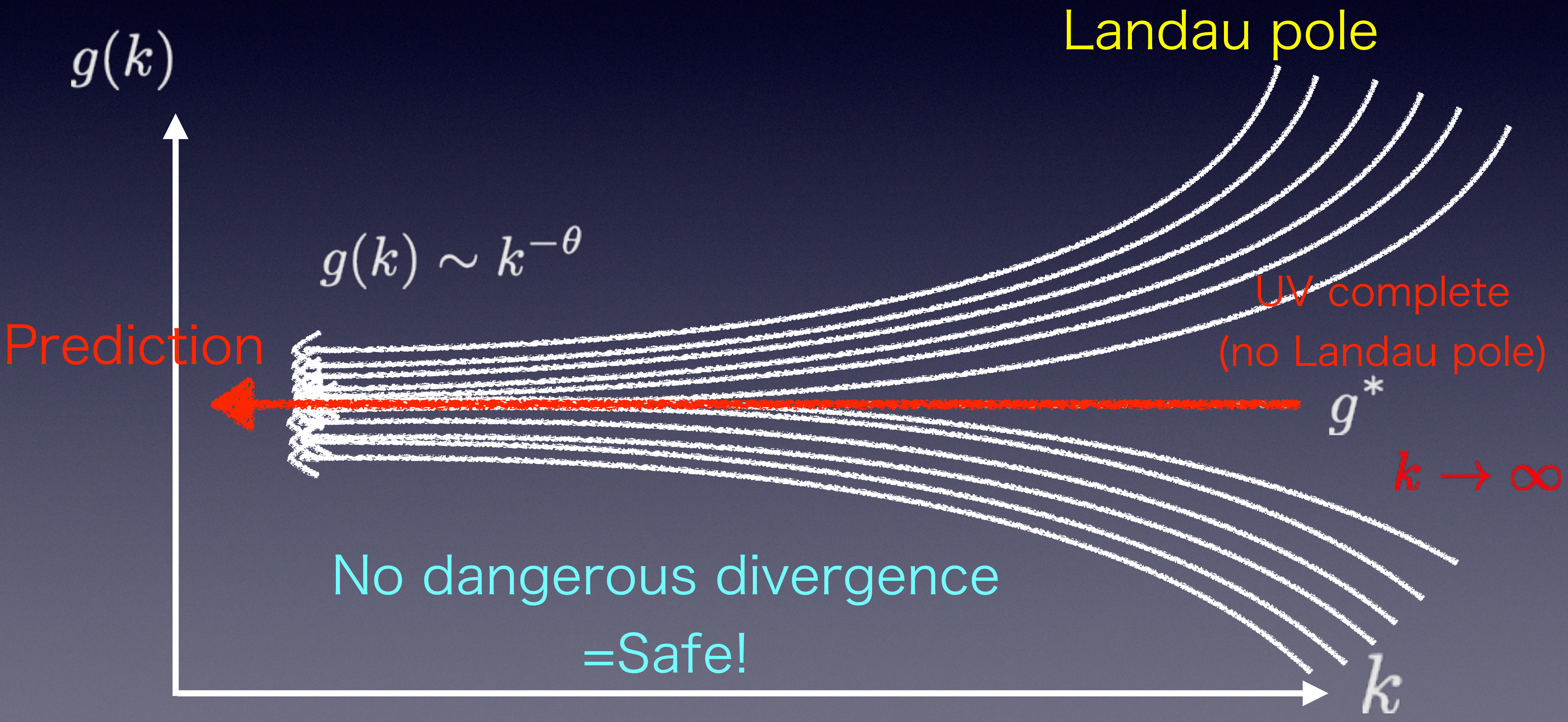


Irrelevant $\theta < 0$

- Predictable parameter



Gravitational system

- Effective action for pure gravity

$$\Gamma_k = \int \mathrm{d}^4x\sqrt{g}\left[V - \frac{M_\mathrm{p}^2}{2}R + CR^2 - DC_{\mu\nu\rho\sigma}^2 + \cdots\right]$$

Gravitational system

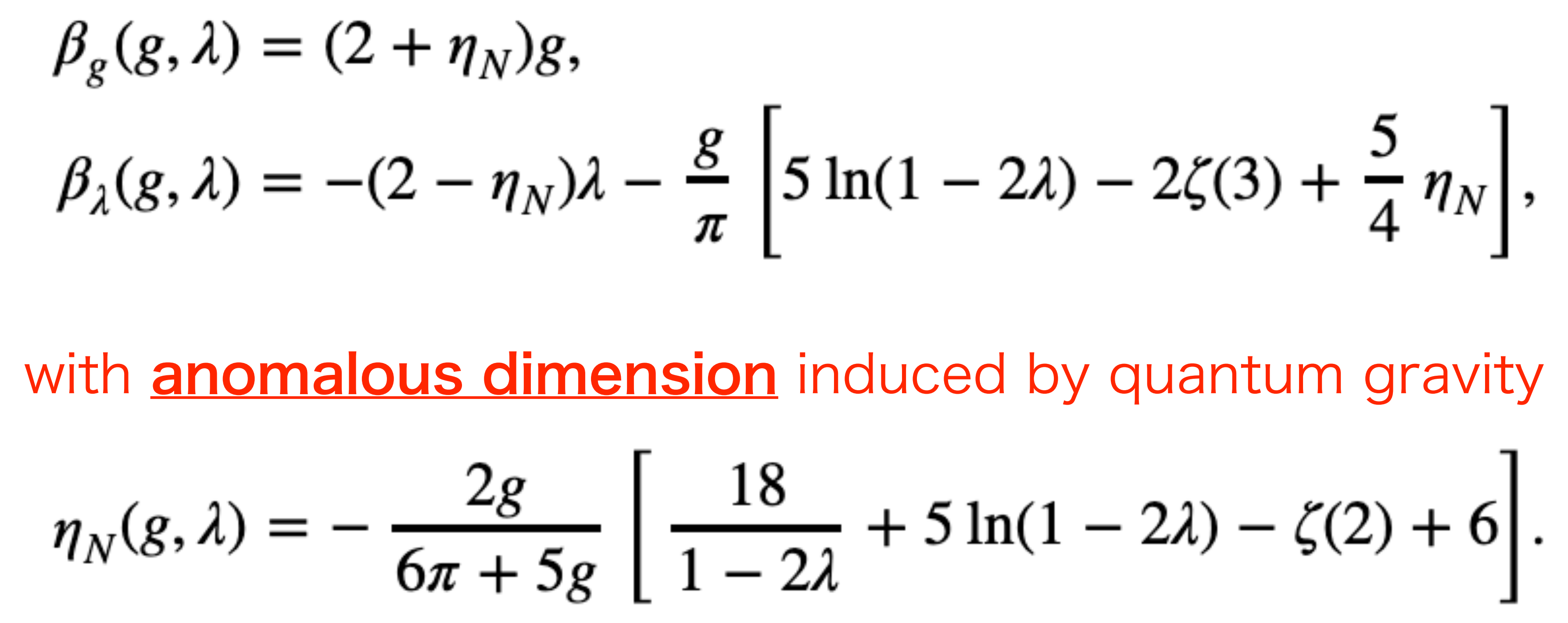
- Effective action for pure gravity

Truncate

$$\Gamma_k = \int \mathrm{d}^4x\sqrt{g}\left[V - \frac{M_\mathrm{p}^2}{2}R + CR^2 \cancel{- DC^2_{\mu\nu\rho\sigma}} + \cdots\right]$$

Einstein-Hilbert truncation

e.g. scholarpedia



$$\beta_g(g,\lambda) = (2+\eta_N)g,$$

$$\beta_\lambda(g,\lambda) = -(2-\eta_N)\lambda - \frac{g}{\pi}\left[5\ln(1-2\lambda) - 2\zeta(3) + \frac{5}{4}\eta_N\right],$$

with **anomalous dimension** induced by quantum gravity

$$\eta_N(g,\lambda) = -\frac{2g}{6\pi+5g}\left[\frac{18}{1-2\lambda} + 5\ln(1-2\lambda) - \zeta(2) + 6\right].$$

$$g_N = \frac{k^2}{8\pi M_\mathrm{p}^2} = k^2 G_N$$

$$\lambda = \frac{V}{8\pi k^2 M_\mathrm{p}^2}$$

Gravitational system

- Fixed point

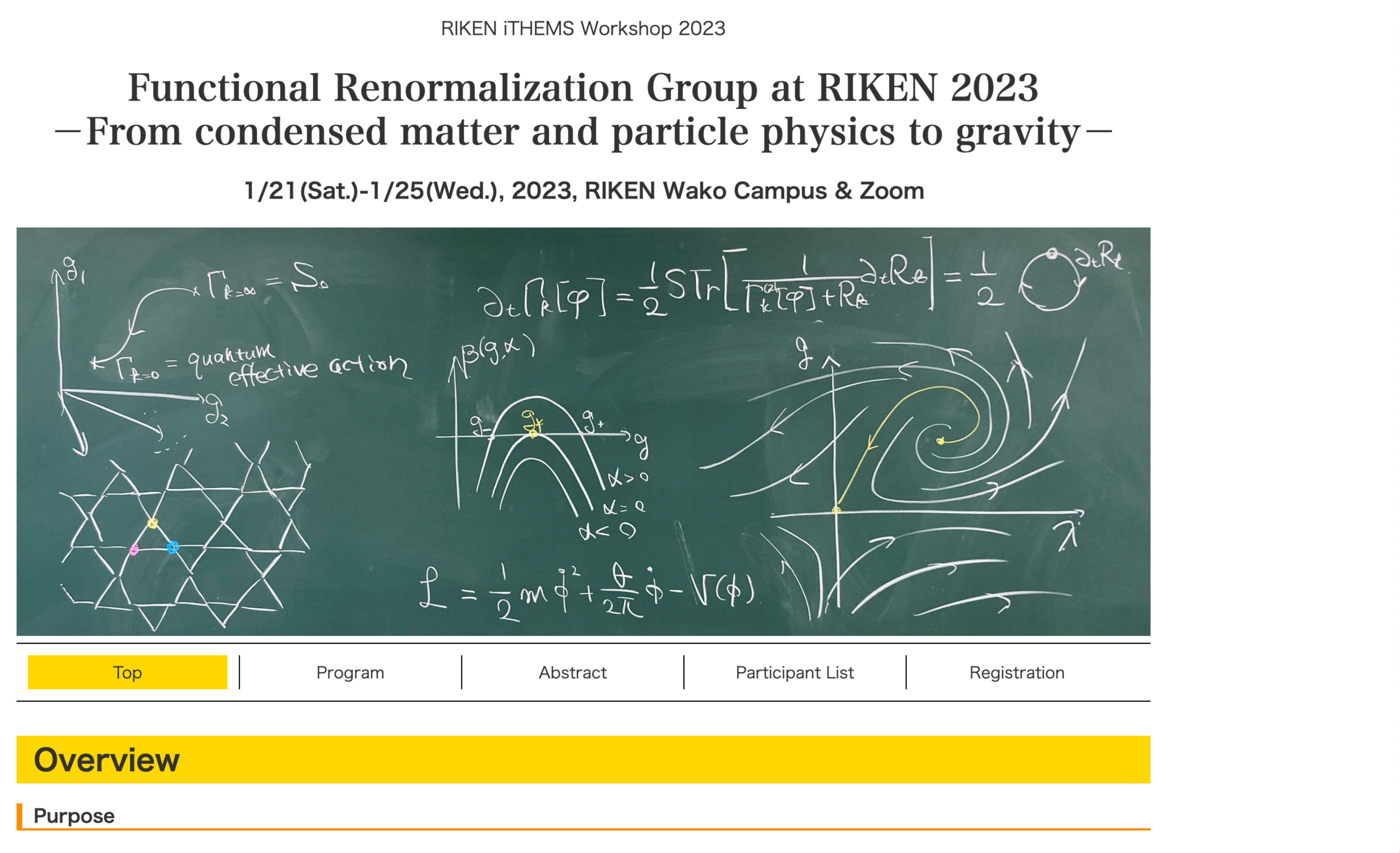
$$\beta_g = \beta_\lambda = 0 \quad \Longrightarrow \quad (g_*, \lambda_*) = (0, 0),\ \ (0.378, 0.340)$$

- Critical exponents

$$\theta_i = -\mathrm{eig} \begin{pmatrix} \frac{\partial \beta_g}{\partial g} & \frac{\partial \beta_g}{\partial \lambda} \\ \frac{\partial \beta_\lambda}{\partial g} & \frac{\partial \beta_\lambda}{\partial \lambda} \end{pmatrix} \Bigg|_{g=g_*, \lambda=\lambda_*}$$

$$(\theta_g, \theta_\lambda) = (-2, 2),\ \ (2.141 + 3.438i, 2.141 - 3.438i)$$

Gravitational system



RIKEN iTHEMS Workshop 2023

Functional Renormalization Group at RIKEN 2023
—From condensed matter and particle physics to gravity—

1/21(Sat.)-1/25(Wed.), 2023, RIKEN Wako Campus & Zoom

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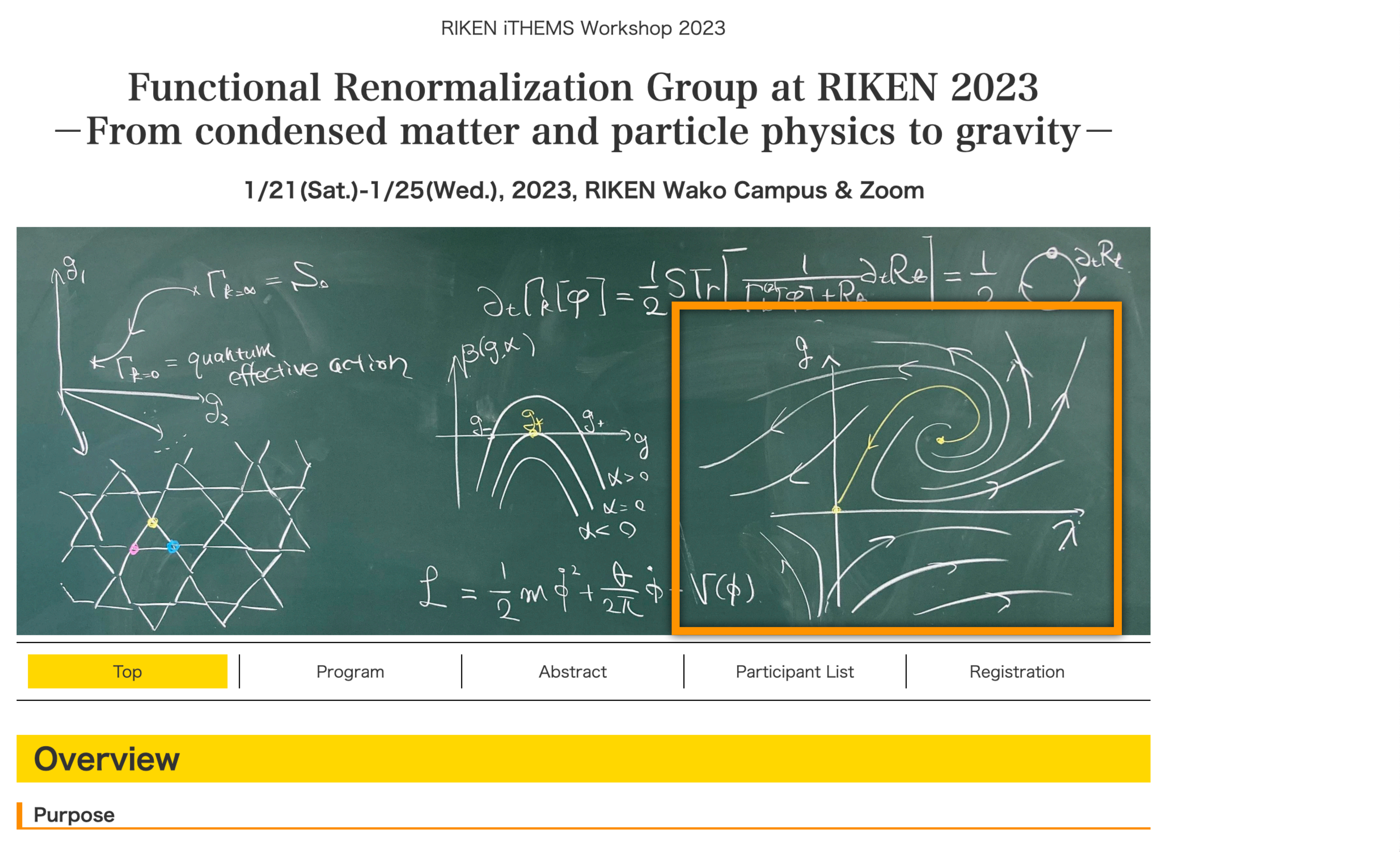
Overview

Purpose

Functional renormalization group (FRG) is a powerful theoretical tool to investigate physical systems described by field theory. Its application

Gravitational system



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Purpose

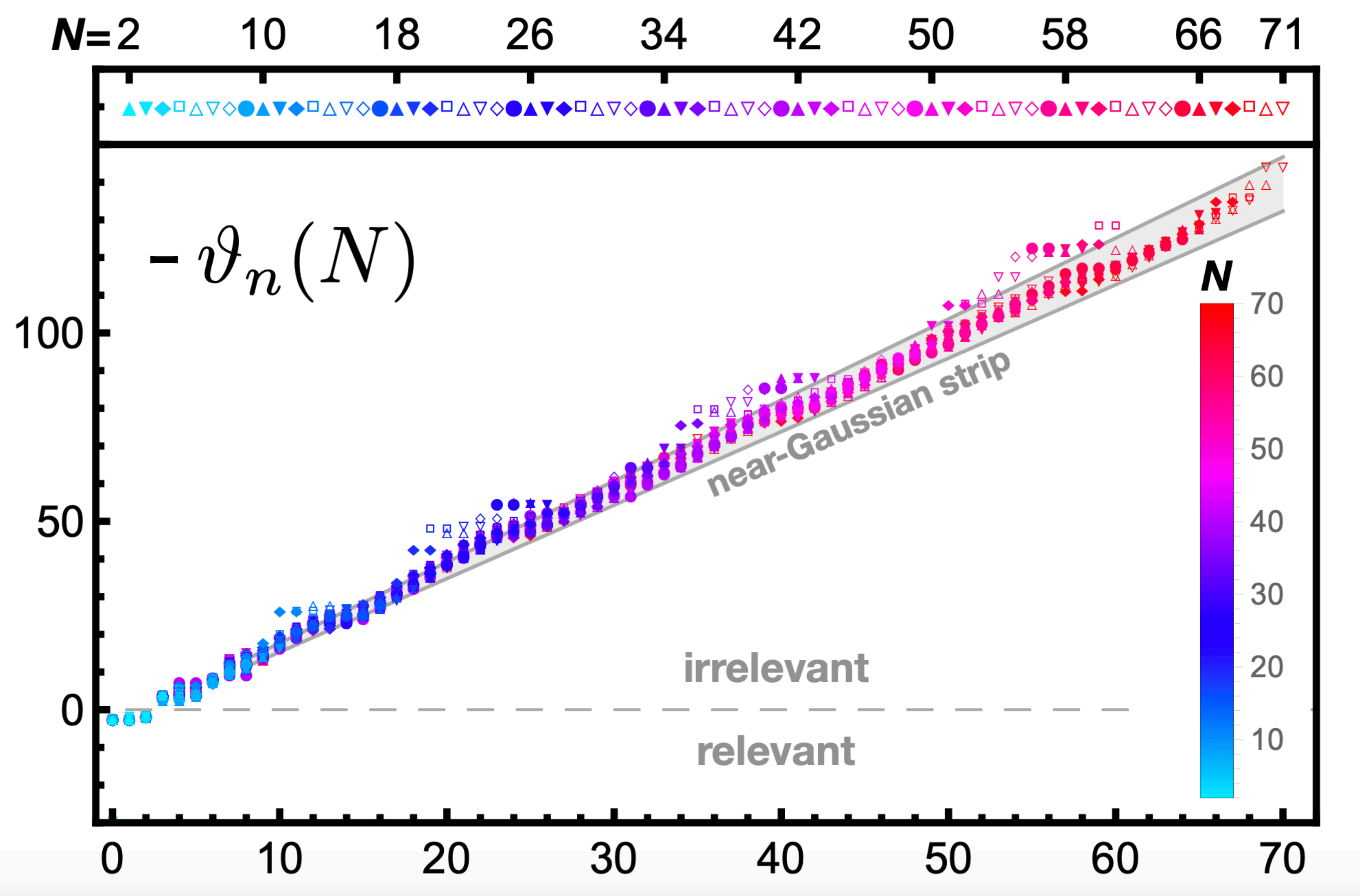
Functional renormalization group (FRG) is a powerful theoretical tool to investigate physical systems described by field theory. Its application

Higher derivative truncation : f(R) truncation

- f(R) truncation (pure gravity)

Phys.Rev. D99 (2019) no.12, 126015

$$\Gamma_k = \int d^4x\sqrt{g}\,f(R) = \int d^4x\sqrt{g}\left[g_0 + g_1 R + g_2 R^2 + g_3 R^3 + \cdots\right]$$



Up to R^{71}

Finite number of relevant couplings =Renormalizalbe!

3 free parameters?

Higher derivative truncation : R2+C2 truncation

- Inclusion of higher dimensional operators

S.Saswato, C. Wetterich, **MY**, JHEP 03 (2022) 130

$$\Gamma_k = \int \mathrm{d}^4x\sqrt{g}\left[V - \frac{M_\mathrm{p}^2}{2}R + CR^2 - DC_{\mu\nu\rho\sigma}^2\right]$$

- Asymptotically safe fixed point

$$v_* = (V/k^4)_* \neq 0 \qquad w_* = (M_\mathrm{p}^2/(2k^2))_* \neq 0 \qquad C_* \neq 0 \qquad D_* \neq 0$$

Graviton propagator: $G_h \sim \dfrac{1}{-v_* + w_* p^2 + D_* p^4}$

- Asymptotically free fixed point

$$v_* = (V/k^4)_* \neq 0 \qquad w_* = (M_\mathrm{p}^2/(2k^2))_* \neq 0 \qquad C_*^{-1} = D_*^{-1} = 0$$

Graviton propagator: $G_h \sim \dfrac{1}{D_* p^4}$

Higher derivative truncation : R²+C² truncation

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$$\Gamma_k = \int \mathrm{d}^4x\sqrt{g}\left[V - \frac{M_\mathrm{p}^2}{2}R + CR^2 - DC^2_{\mu\nu\rho\sigma}\right]$$

$$\text{coupling} \sim k^{-\theta}$$

- Asymptotically safe fixed point

$v_* = (V/k^4)_* \neq 0$ $w_* = (M_\mathrm{p}^2/(2k^2))_* \neq 0$ $C_* \neq 0$ $D_* \neq 0$

$\theta_1 = 3.1$ $\theta_2 = 2.4$ $\theta_3 = 10.9$ $\theta_4 = -88.1$

- Asymptotically free fixed point

$v_* = (V/k^4)_* \neq 0$ $w_* = (M_\mathrm{p}^2/(2k^2))_* \neq 0$ $C_*^{-1} = D_*^{-1} = 0$

$\theta_1 = 4$ $\theta_2 = 2$ $\theta_3 = \theta_4 = 0$

Contents

- Asymptotically safe gravity:
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Is asymptotically safe theory ultimate?

An Example of asymptotically safe theories

- Non-linear σ model in 3 dim.
  - Scalar theory with a field constraint $\langle \phi^i \phi^j \rangle = f_\pi^2 \delta^{ij}$
  - Symmetry breaking *O*(*N*) → *O*(*N*-1) in the linear σ model.

$$\phi^i = (\sigma, \pi^1, \ldots, \pi^{N-1}) \qquad \langle \sigma \rangle = f_\pi$$

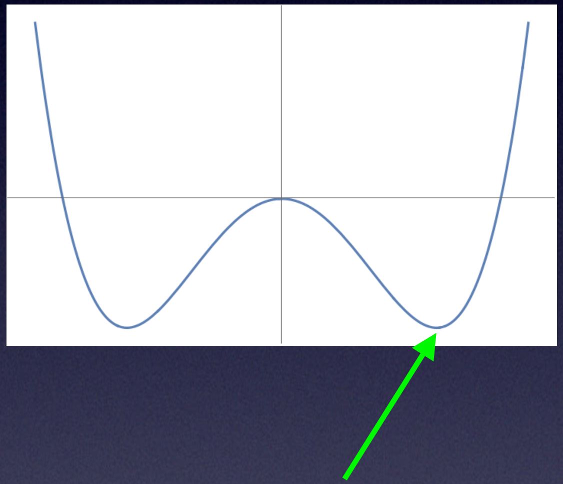
  - Describes dynamics of massless NG bosons (pions). $S[\pi^i]$
  - Perturbatively ***non***-renormalizable

An Example of asymptotically safe theories

- Linear σ model

$$S = \int d^3x \left[\frac{1}{2}(\partial_\mu \phi^i)^2 - \frac{m^2}{2}(\phi^i)^2 - \frac{\lambda}{4}(\phi^i)^4\right]$$

$$\phi^i = (\sigma, \pi^1, \ldots, \pi^{N-1})$$



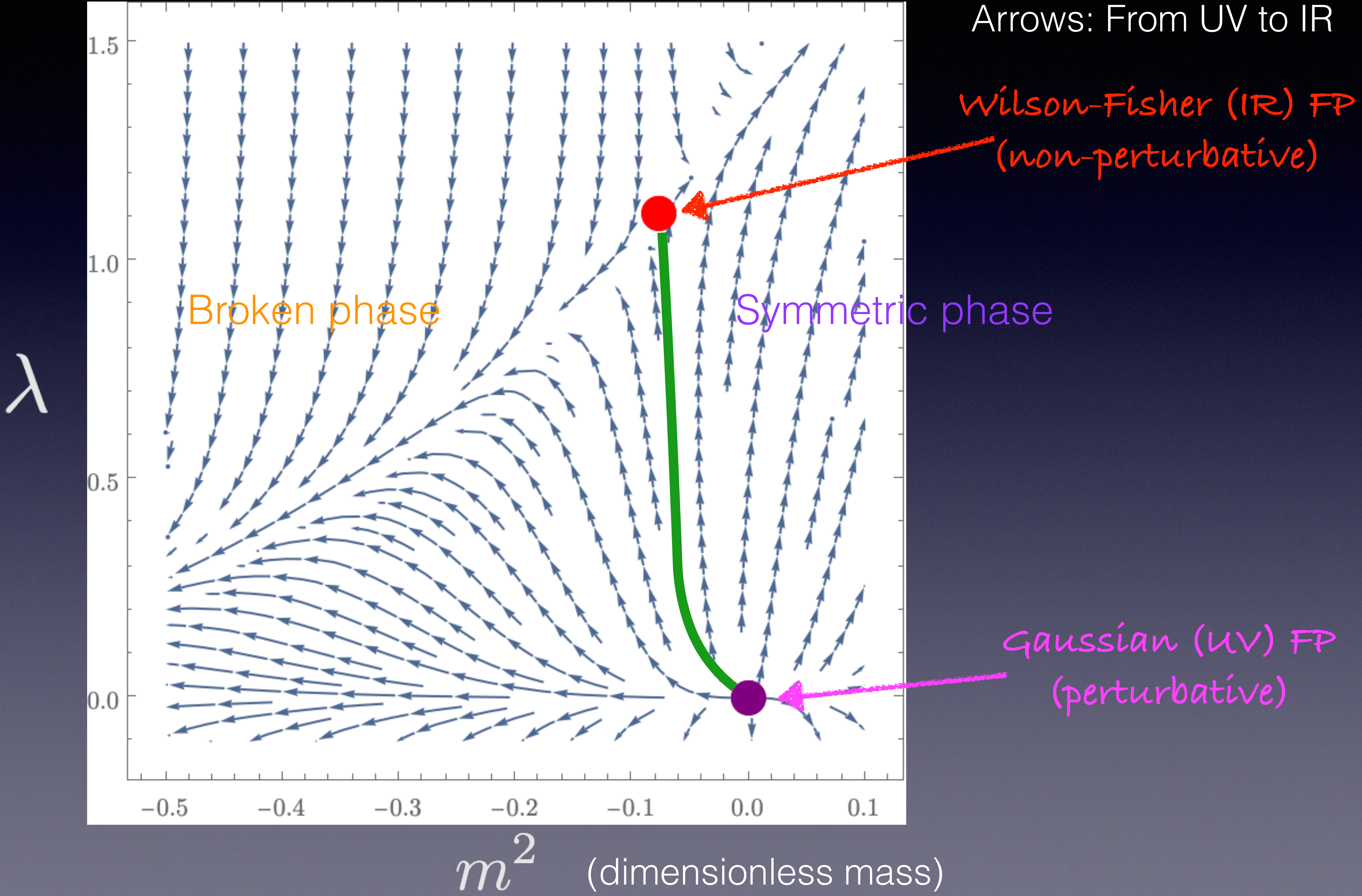
$$f_\pi = \langle \phi \rangle = \sqrt{-2m^2/\lambda}$$

$$\langle \phi^i \phi^j \rangle = f_\pi^2 \delta^{ij}$$

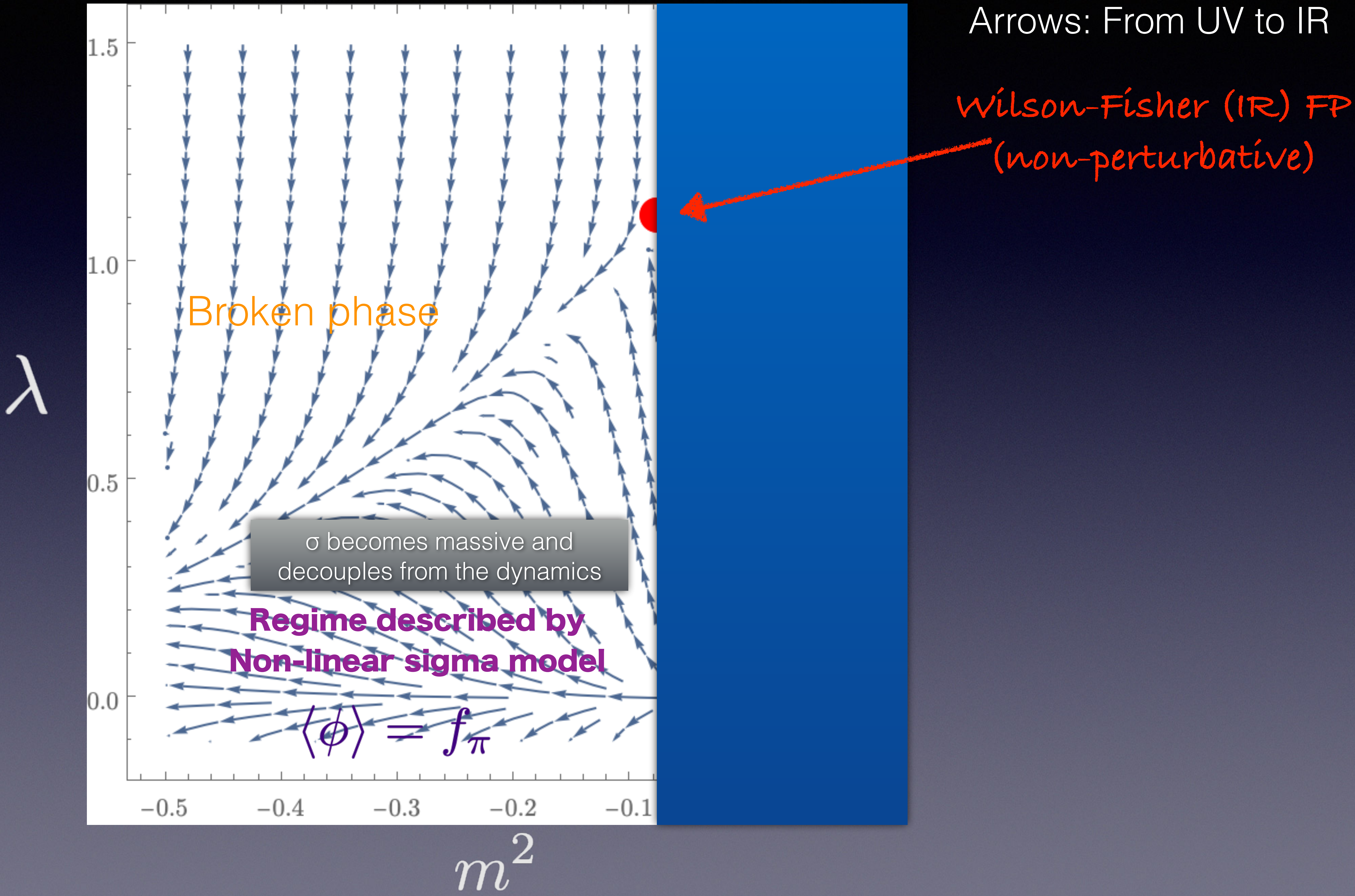
- Non-linear σ model

$$S = \int d^3x \left[\frac{f_\pi^2}{2}(\partial_\mu \pi^i)^2 + \frac{a}{2}(\pi^i \partial_\mu \pi^i)^2 + \cdots\right]$$

Phase diagram of 3d linear σ model

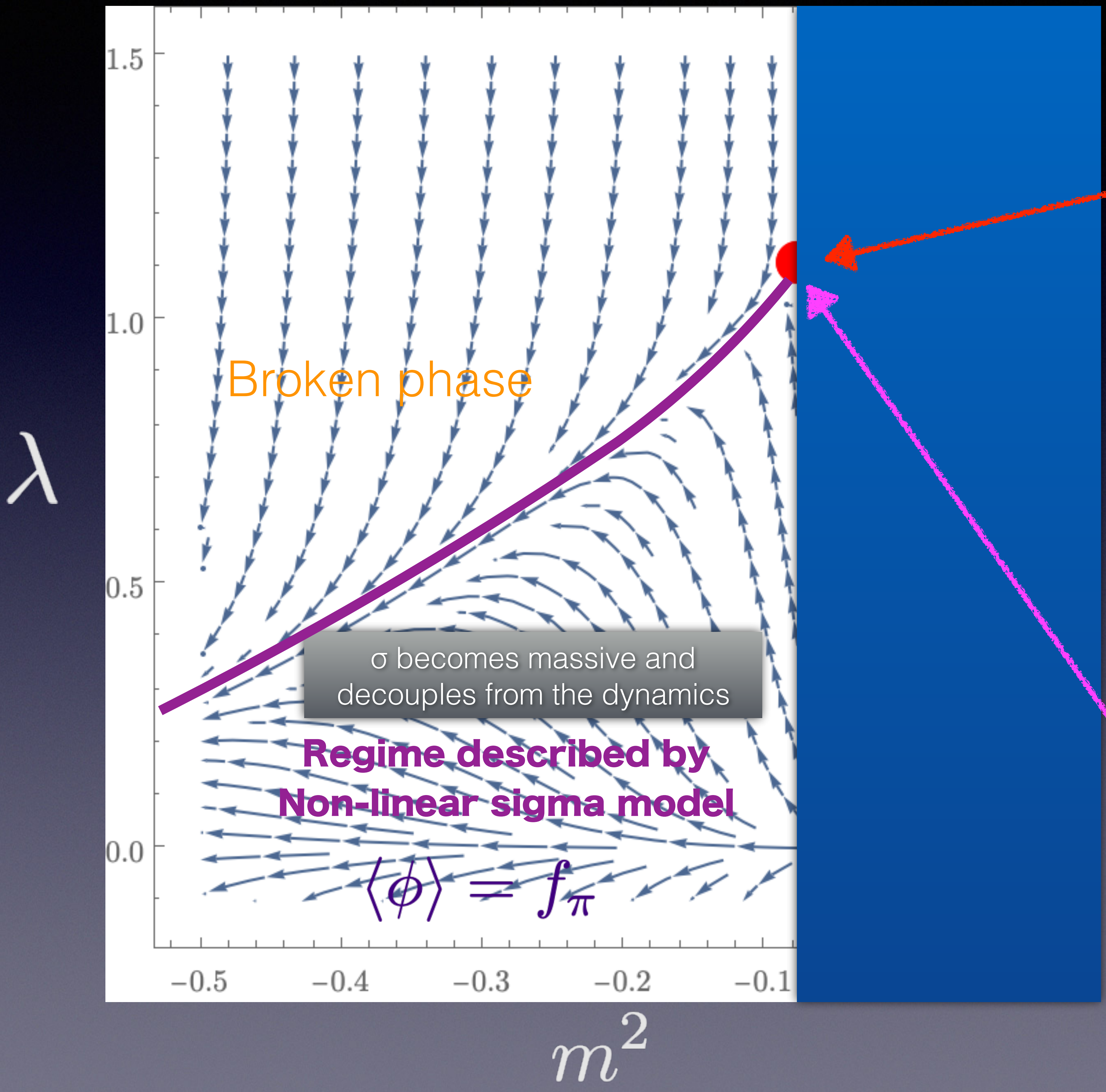


Phase diagram of 3d linear σ model



Arrows: From UV to IR

Phase diagram of 3d linear σ model

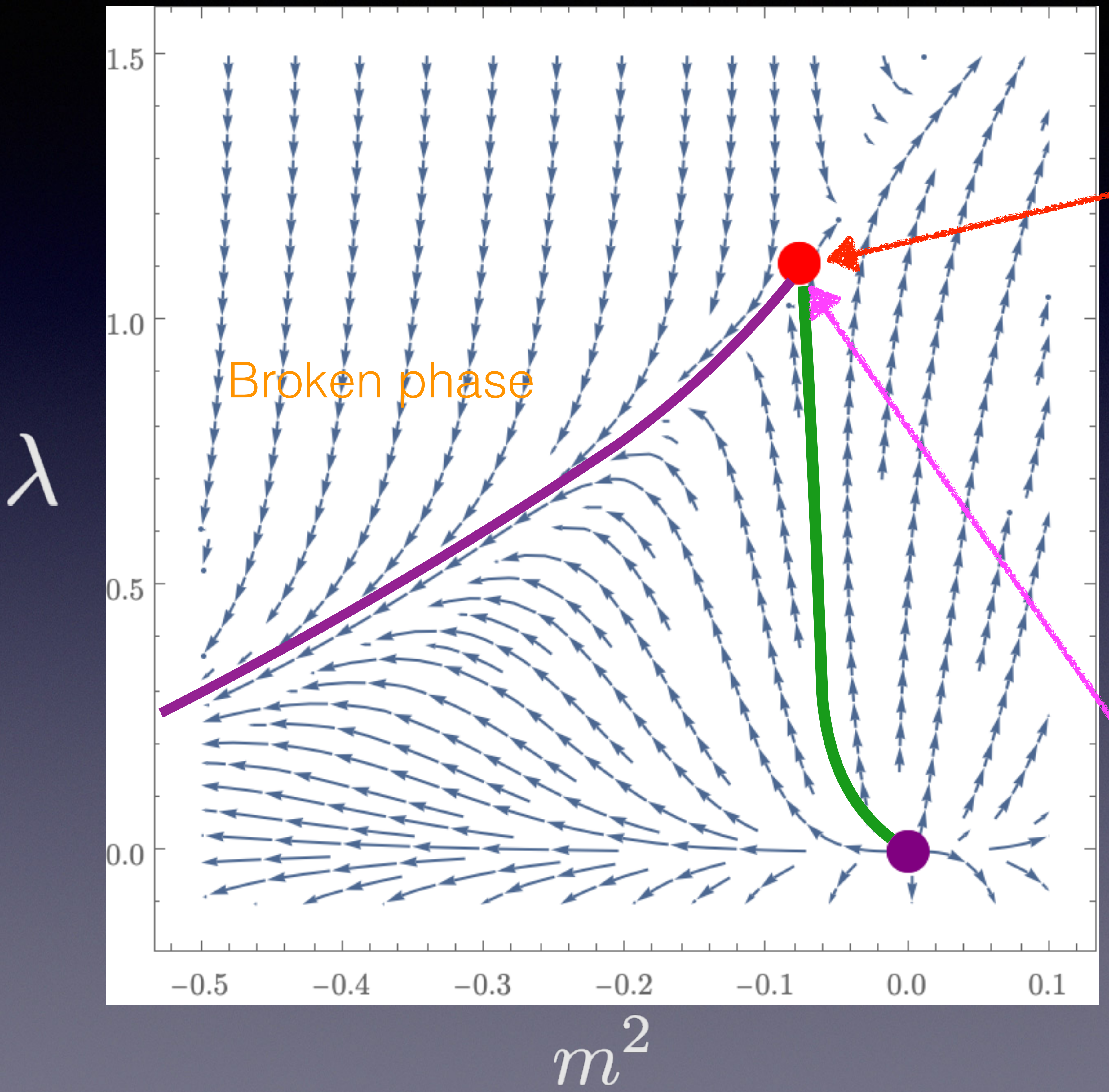


Arrows: From UV to IR

Wilson-Fisher <u>(IR)</u> FP
(non-perturbative)

Non-trivial <u>UV</u> FP
(non-perturbative)

Phase diagram of 3d linear σ model



Arrows: From UV to IR

Wilson-Fisher (IR) FP
(non-perturbative)

linear σ model

Same universality class

non-linear σ model

Non-trivial UV FP
(non-perturbative)

To summarize

Non-linear σ model in 3 dim O(N-1)

- Perturbatively non-renormalizable
- Asymptotically safe (UV FP)
- Constraint on fields

$$\langle \phi^i \phi^j \rangle = f_\pi^2 \delta^{ij}$$

Same universality class

O(N) linear σ model in 3 dim

- Perturbatively renormalizable
- Unitary (at Gaussian FP)
- Asymptotically free (Gaussian FP)
- IR fixed point (Wilson-Fisher FP)

Asymptotically safe gravity

- Perturbatively non-renormalizable
- Asymptotically safe (UV FP)
- Constraint on fields

$$g_{\mu\alpha} g^{\alpha\nu} = \delta_\mu^\nu$$

?

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Scalar-gravity system

- Effective action

$$\Gamma_k = \int d^4x\sqrt{g}\left[\frac{Z_\phi}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi) - \frac{F(\phi)}{2}R + \cdots\right]$$

  - Einstein-Hilbert truncation: linear in R
  - Minimal coupling $F(\phi) = M_{\mathrm{p}}^2 = \frac{1}{8\pi G}$
  - Local potential approximation $Z_\phi = 1$

Critical exponent of scalar interactions

- Effective potential

$$V(\phi) = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \cdots$$

canonically relevant

canonically marginal

canonically irrelevant

Gaussian FP: $\theta_m = 2$ $\theta_\lambda = 0$ $\theta_i < 0$

Non-trivial FP:

$\theta_m = 2 - \gamma_m$ $\theta_\lambda = -\gamma_\lambda$

$\theta_i < 0$

$\theta_i > 0$

Gravity-induced anomalous dimension

- RG for scalar couplings

$$\partial_t \tilde{m}^2 = (-2 + A)\tilde{m}^2 - \frac{3\tilde{\lambda}}{32\pi^2}\frac{1}{(1+\tilde{m}^2)^2}$$

$$\partial_t \tilde{\lambda} = A\tilde{\lambda} + \frac{9\tilde{\lambda}^2}{16\pi^2}\frac{1}{(1+\tilde{m}^2)^3}$$

- Anomalous dimension

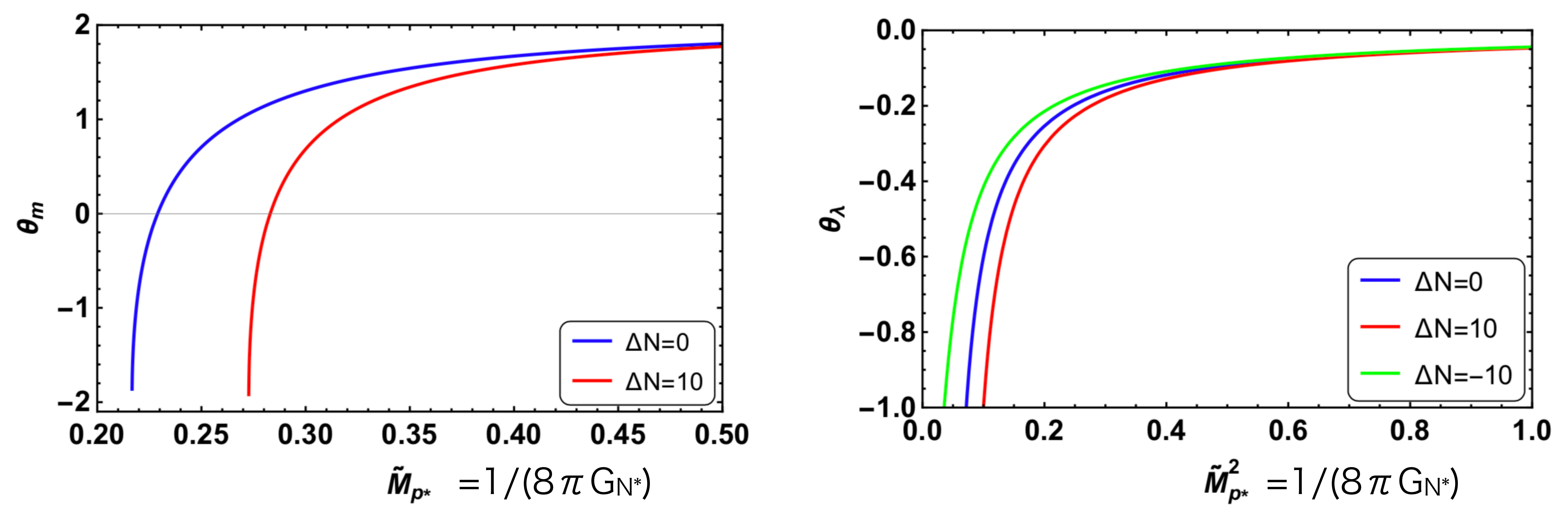
Pawlowski, Reichert, Wetterich, **MY**, Phys.Rev. D99 (2019) no.8, 086010

$$A = \frac{1}{48\pi^2 \tilde{M}_{\rm p}^2}\left[\frac{20}{(1-v_0)^2} + \frac{1}{(1-v_0/4)^2}\right] \qquad v_0 = \frac{2\Lambda_{\rm cc}}{k^2 \tilde{M}_{\rm p}}$$

Critical exponent

$\theta_m \simeq 2 - A$

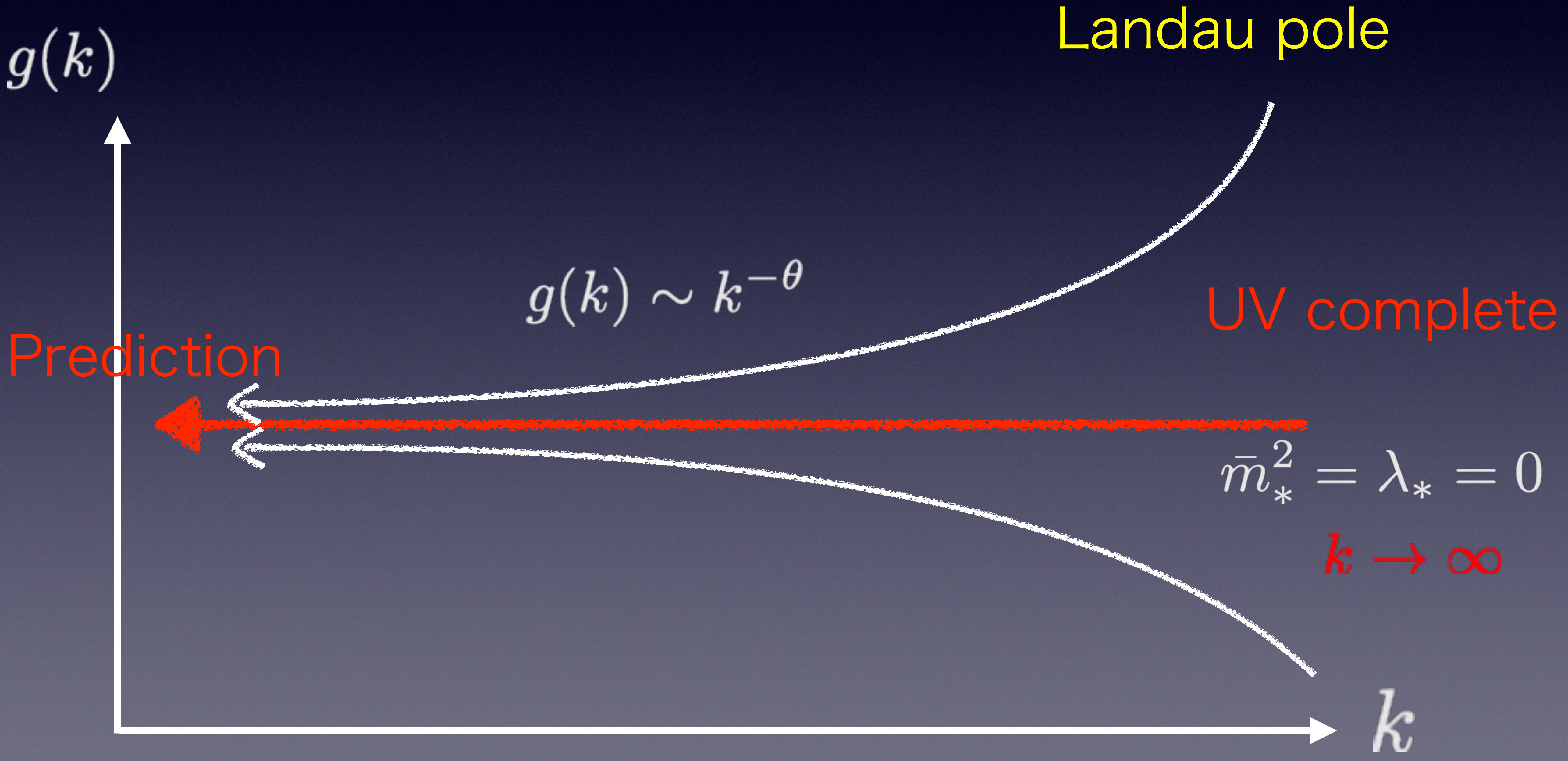
$\theta_\lambda = -\gamma_\lambda = -A$



Gravitational interaction tends to make scalar couplings irrelevant!

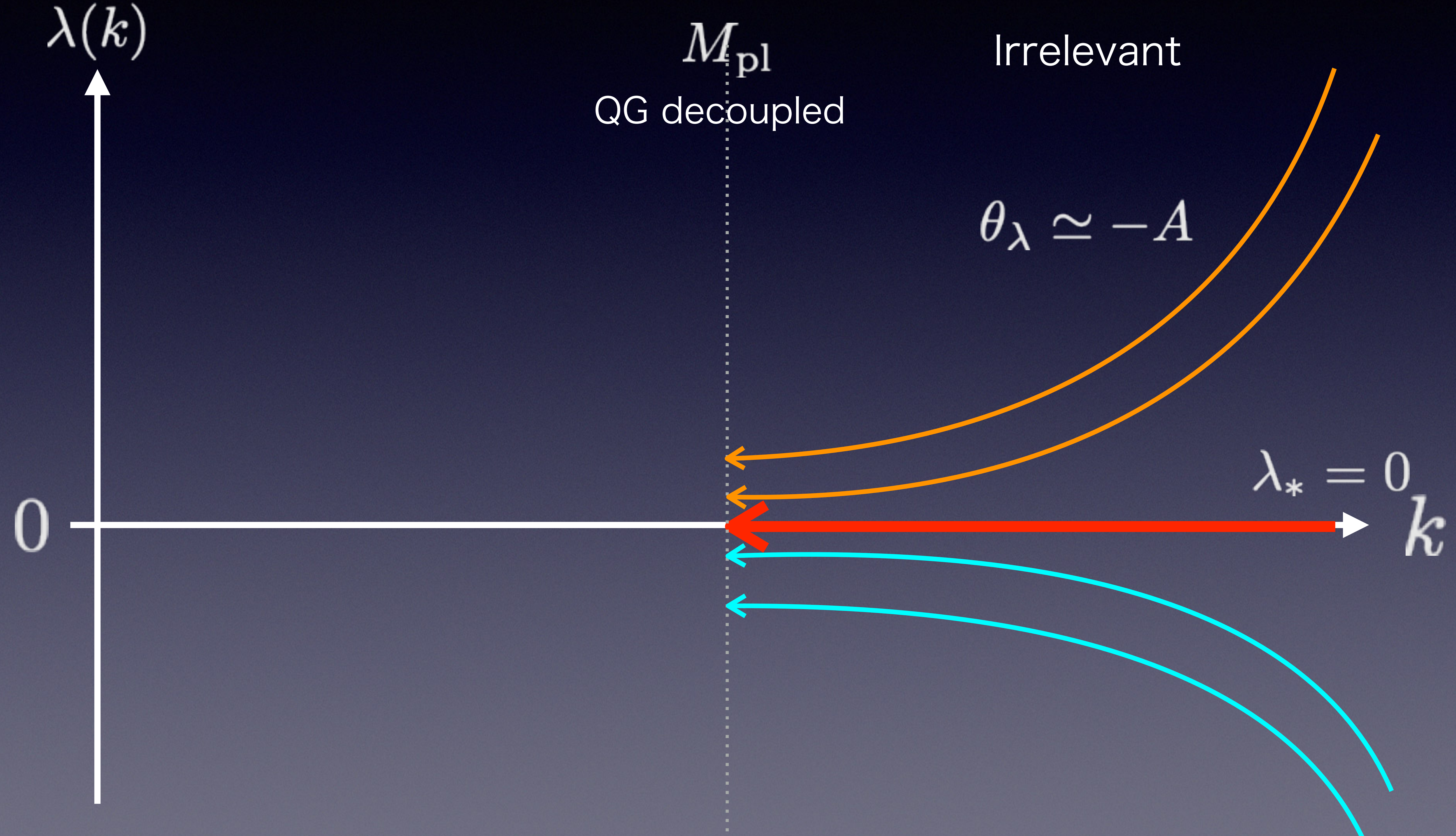
Irrelevant $\theta < 0$

- Predictable



$$\lambda(k) = \lambda_0 \left(\frac{k}{\Lambda}\right)^{-\theta_\lambda}$$

RG flow



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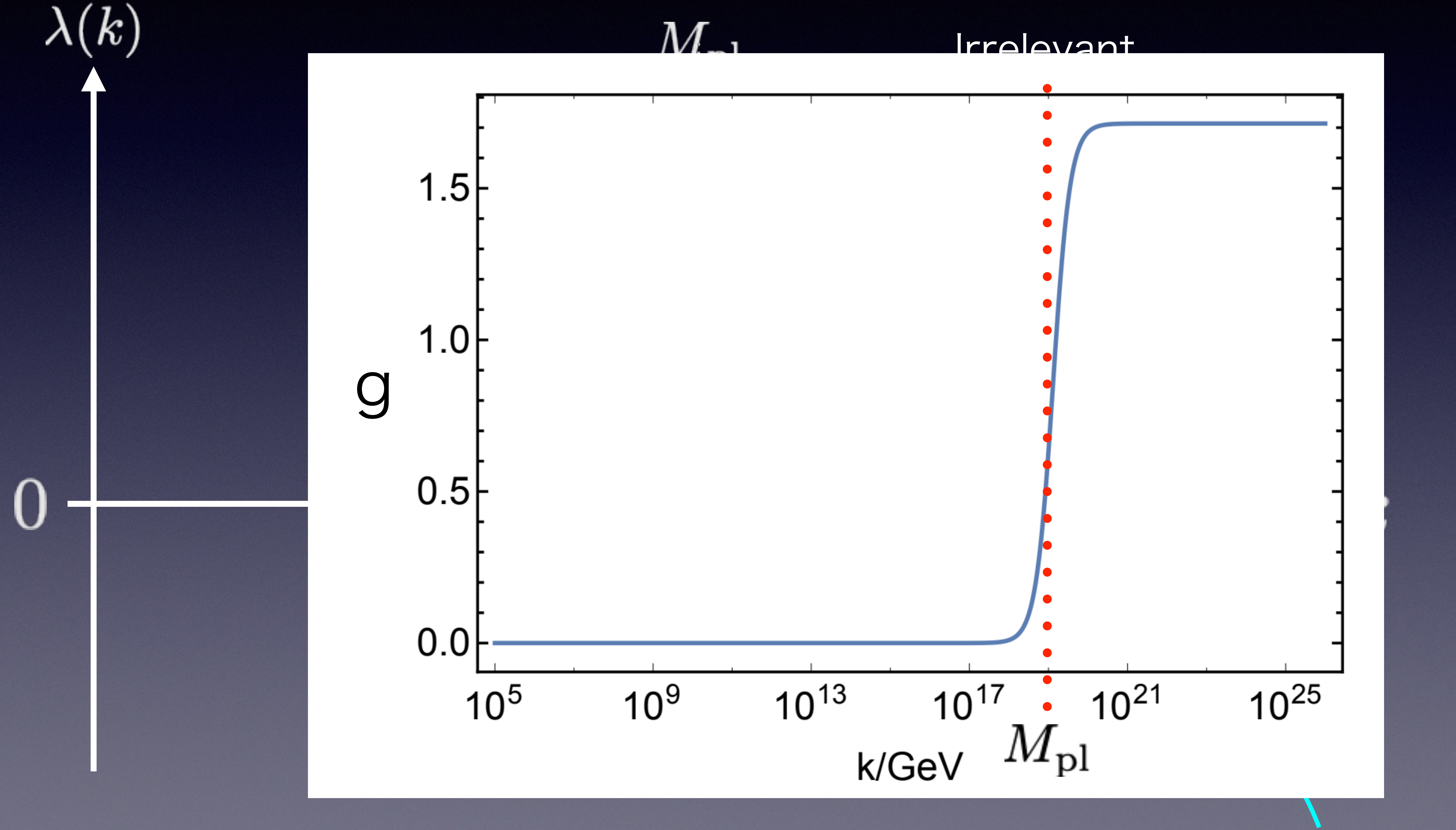

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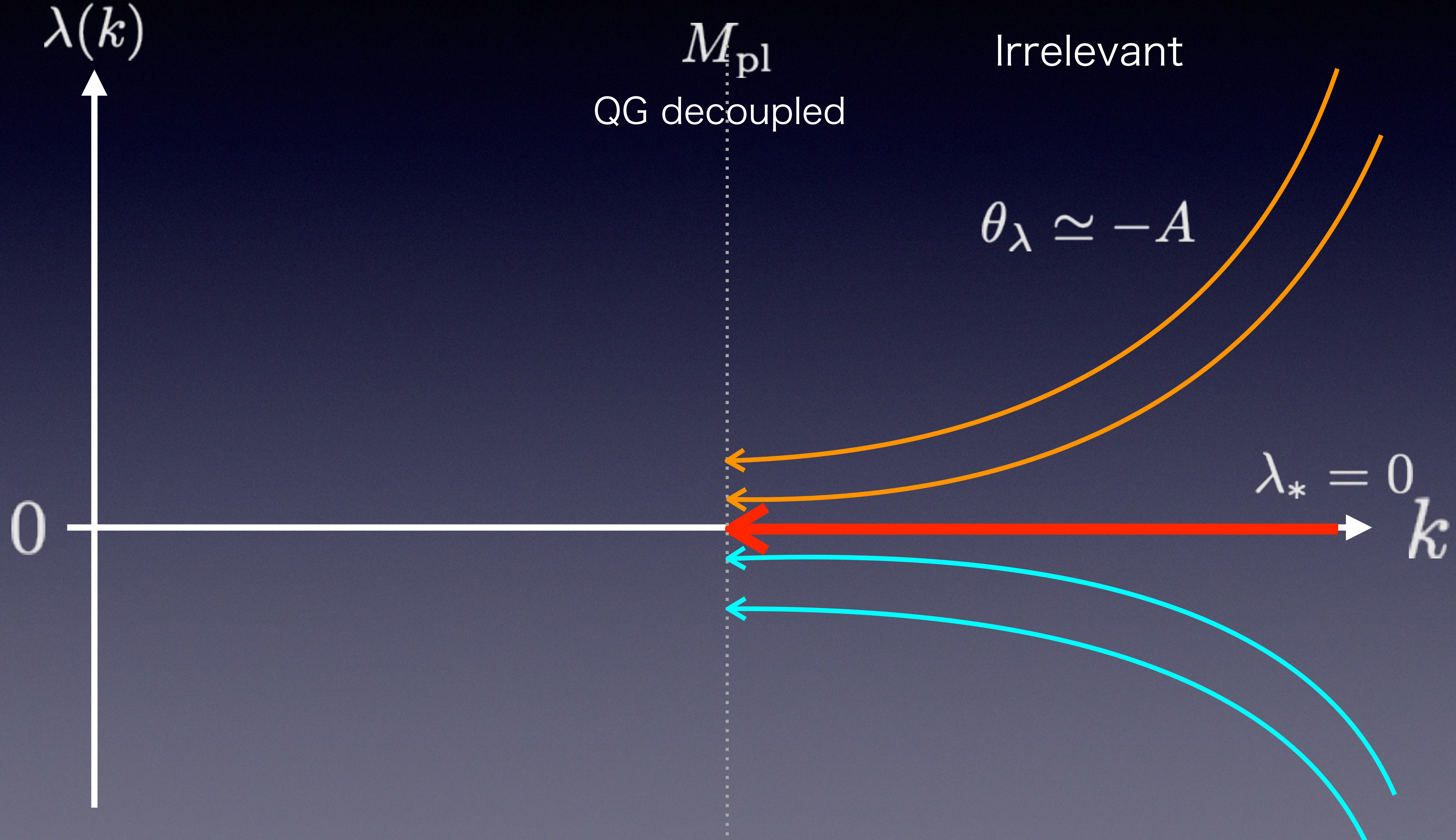

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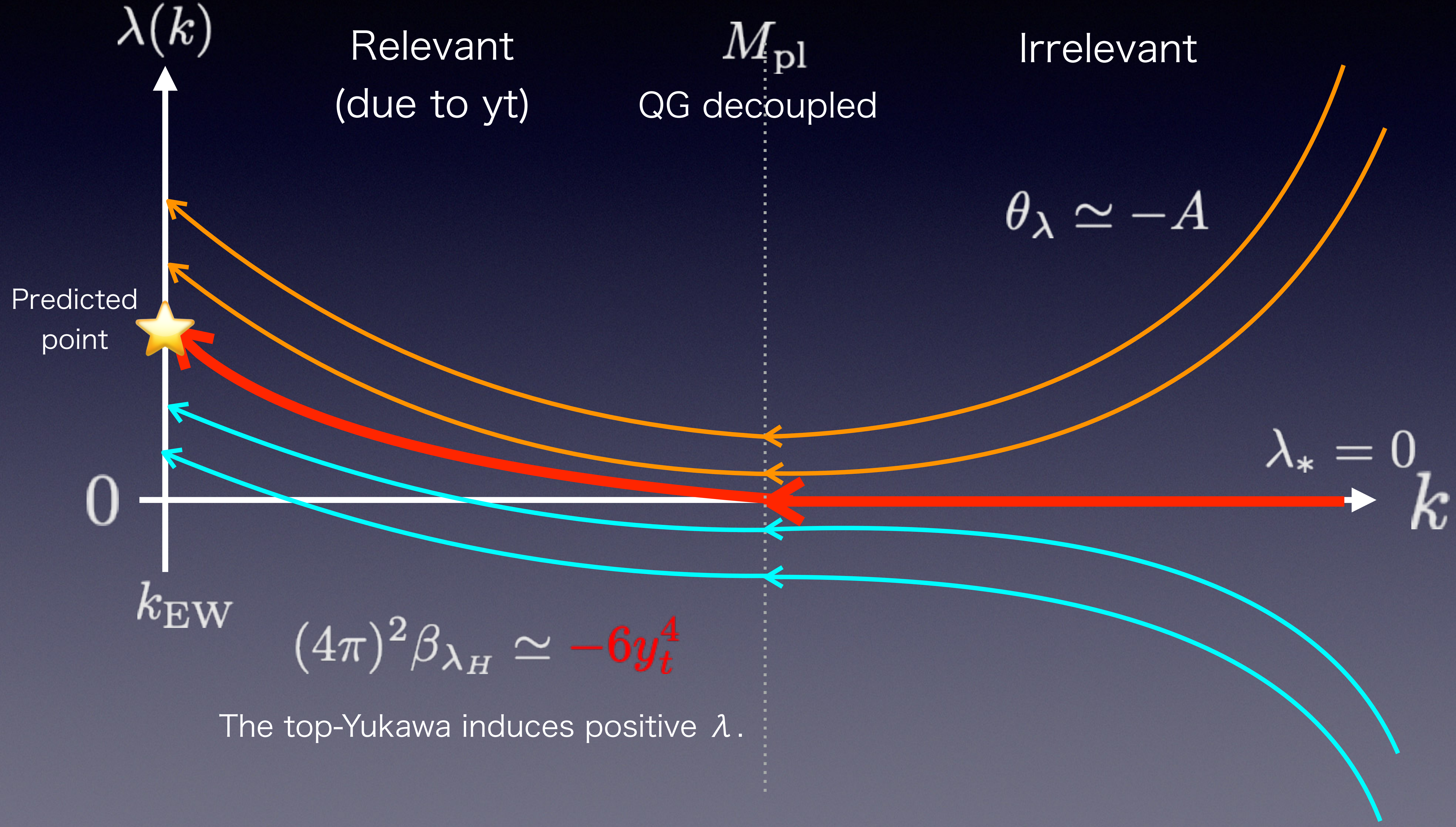
RG flow



$$(4\pi)^2\beta_{\lambda_H} \simeq -6y_t^4$$

The top-Yukawa induces positive λ.

Top quark mass
vs.
Higgs mass

- For m_t=171.3 GeV, m_H=126.5 GeV

Shaposhnikov, Wetterich, Phys.Lett. B683 (2010) 196-200

- For m_t=230 GeV, m_H=233 GeV

- Current experimental results (LHC)

Eur. Phys. J. C 80 (2020) 658; PDG

  - m_t=170.5±0.7 GeV, m_H=125.10±0.14 GeV

Higgs portal interaction

- SM extension including new scalar S

$$V(S,H) = m_H^2 H^\dagger H + m_S^2 S^\dagger S + \lambda_H \left(H^\dagger H\right)^2 + \lambda_{HS} \left(H^\dagger H\right) S^\dagger S + \lambda_S \left(S^\dagger S\right)^2$$

- All couplings irrelevant:

Eichhorn, Yuta.Hamada, Lumma, **M.Y.**, Phys.Rev. D97 (2018) no.8, 086004

$$m_H^2 = m_S^2 = 0$$

$$\lambda_H = \lambda_{HS} = \lambda_S = 0$$

at $k = M_{\mathrm{pl}}$

- The potential is flat above the Planck scale. (Flatland)

- This becomes strong constrains on extensions of the SM.

Possible extension of the SM

- The boundary condition at the Planck scale:

$$\lambda_H = \lambda_{HS} = \lambda_S = 0 \qquad \text{at} \quad k = M_{\rm pl}$$

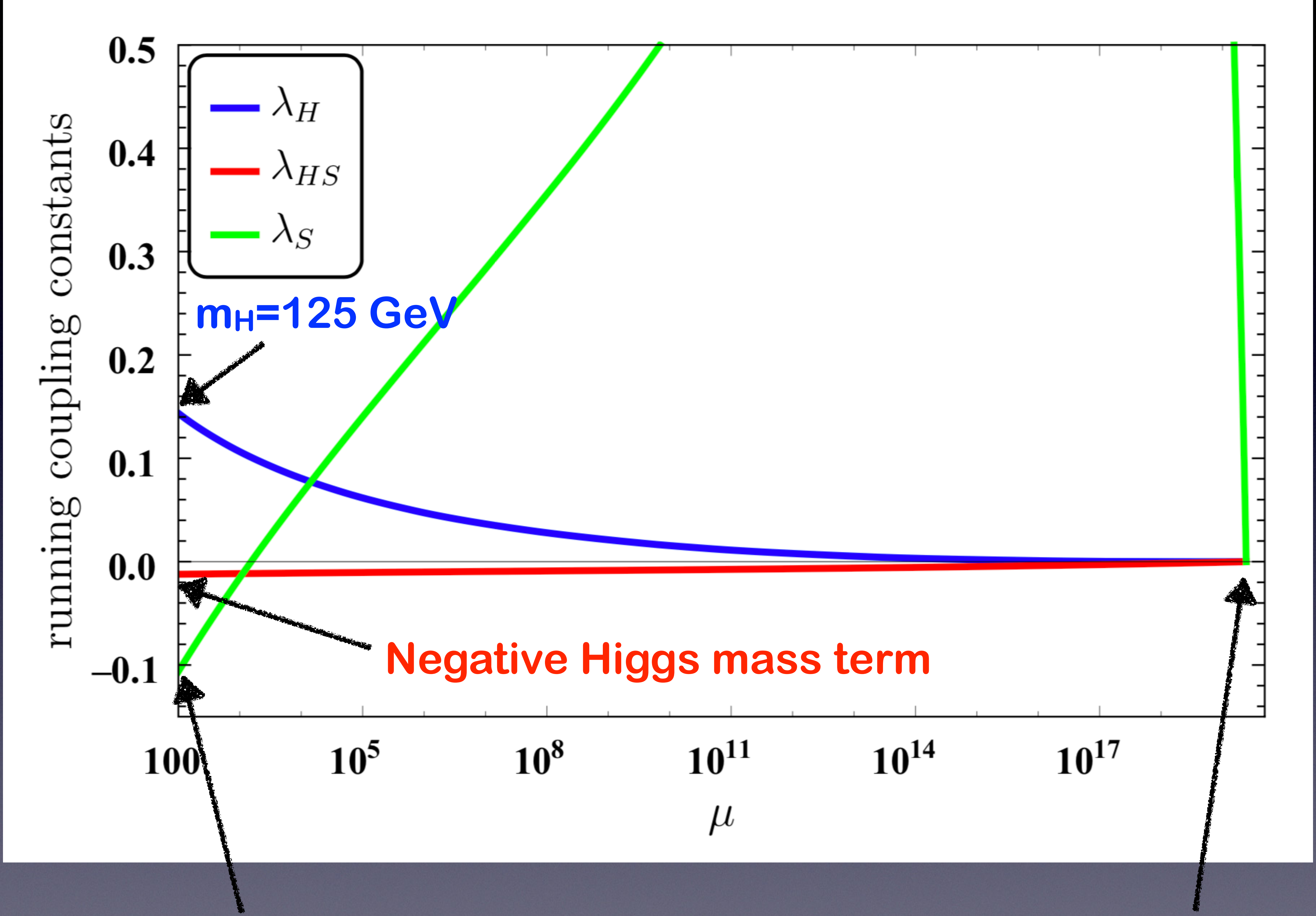
- To generate finite values in low energy
    - Additional Majorana fermions χs and U(1) gauge field X_μ

$$k\frac{d\lambda_S}{dk} = -n_\chi y_\chi^4 + n_X {g_X}^4 + \cdots$$

Kinetic mixing

$$k\frac{d\lambda_{HS}}{dk} = n_{\rm mix}\ g_{\rm mix}^2\ g_X^2 + \cdots$$

$$X_\mu B^\mu$$



m_H=125 GeV

Negative Higgs mass term

Coleman-Weinberg for <S>

Flatland

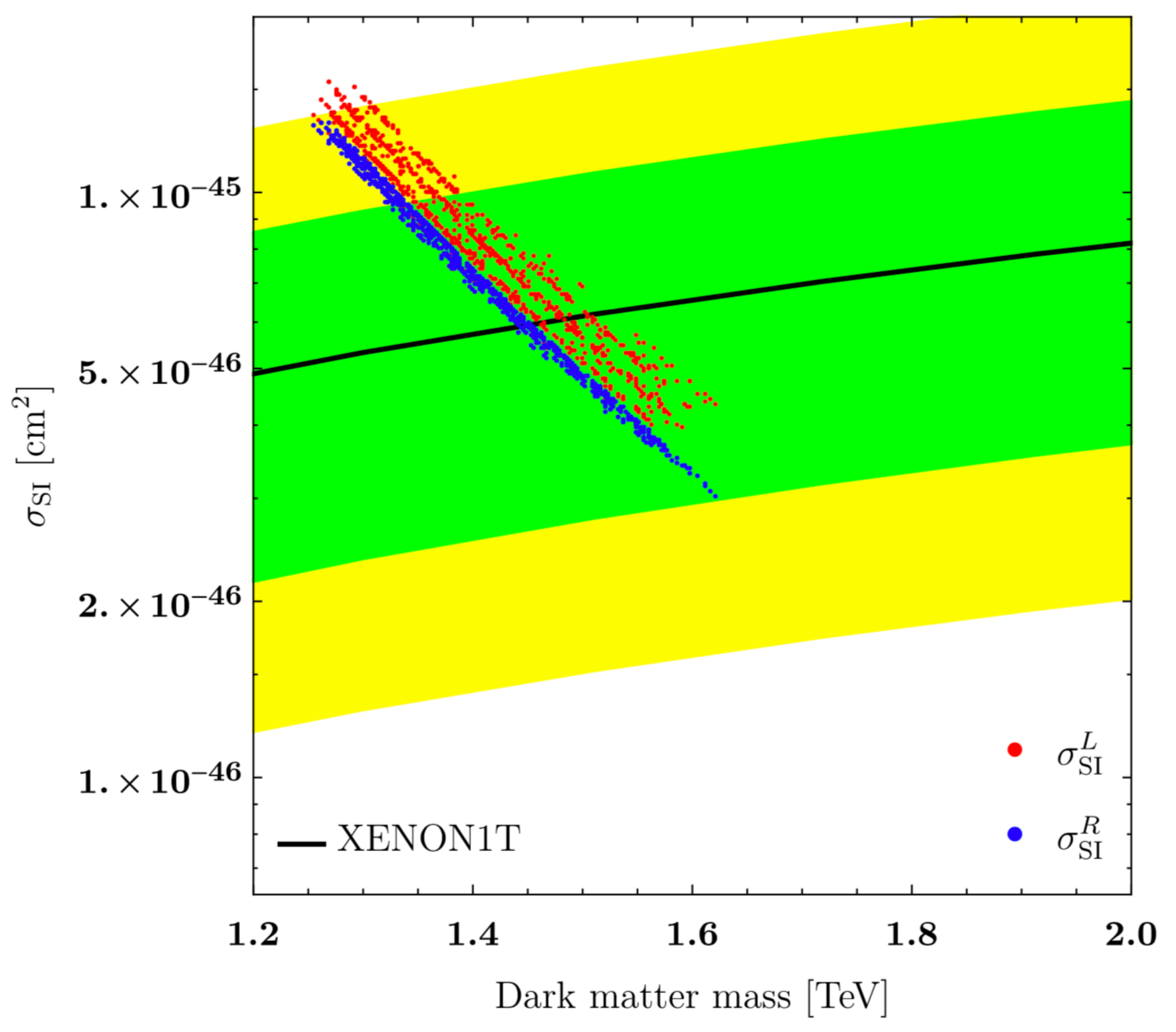
Strong prediction

- 7 parameters (λ_H, λ_{HS}, λ_S, y_L, y_R, g_X, g_{mix})
  ⟹ 1 parameter

  - 6 constraints:
    - Higgs mass: m_H=125 GeV
    - Electroweak vacuum: v_H=246 GeV
    - Majorana fermions as dark matters: $\Omega_{DM}h^2$=0.12
    - Flatland condition:

$$\lambda_H = \lambda_{HS} = \lambda_S = 0 \quad \text{at} \quad k = M_{\rm pl}$$

Prediction to Dark matter physics (Majorana fermionic DM)

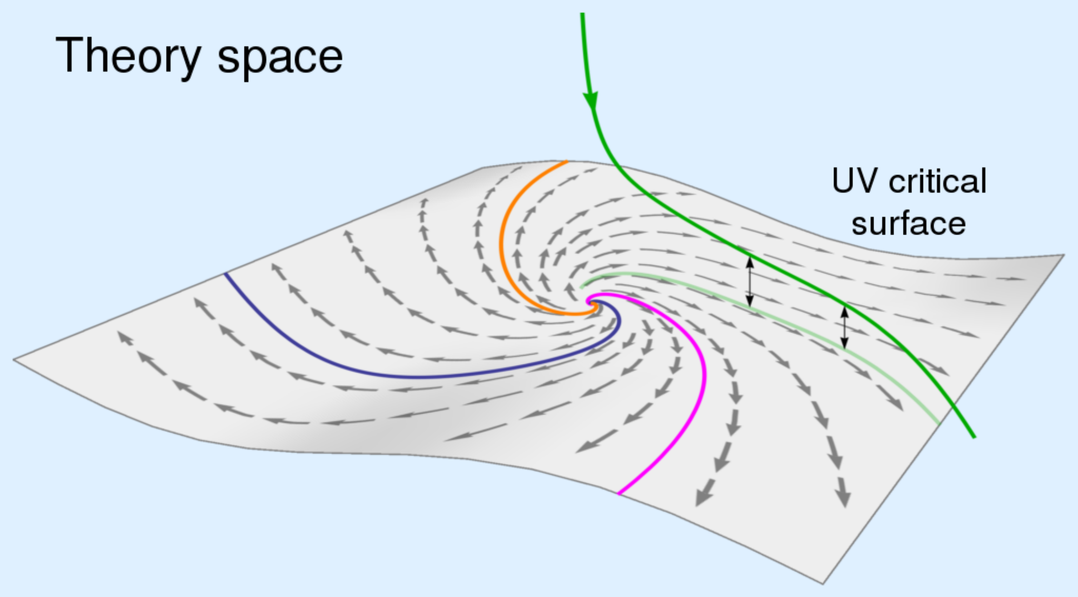


Summary

- Asymptotically safe gravity:
  - Non-perturbatively renormalizable
  - Universality class
  - Indicate the existence of new degrees of freedom.
- Strong constrains on extensions of the SM.

Appendix

Asymptotic safety



- Suggested by S. Weinberg

S. Weinberg, Chap 16 in General Relativity

- Existence of UV fixed point
  - Continuum limit k→∞.
  - UV critical surface (UV complete theory) is defined by relevant operators.
  - Dimension of UV critical surface = number of free parameters.
- Generalization of asymptotic free

Asymptotic freedom vs safety

Asymptotic freedom

$$\beta(g) = -\left(0 + \beta_0 g^2\right) g$$

canonical scaling

anomalous scaling

Asymptotic safety

$$\beta(g) = -\left(-2 + \beta_0 g^2\right) g$$

canonical scaling

anomalous scaling

$$\beta(g) = 0$$

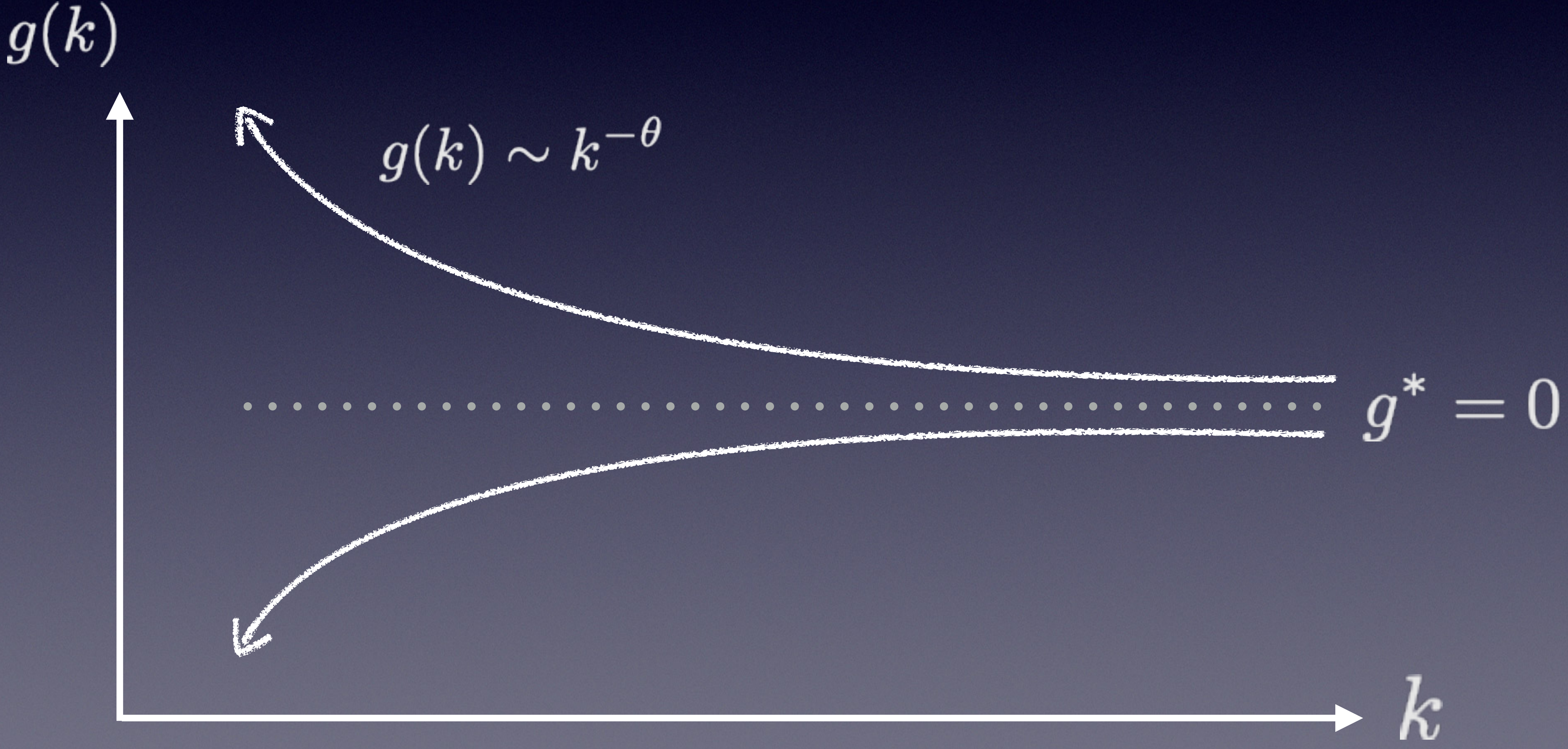
$$g_* = 0 \qquad g^2 \sim \beta_0 g_*^2 \log k$$

$$g_* = 0 \qquad g^2 \sim \beta_0 g_*^2 \log k$$

$$g_* = \sqrt{2/\beta_0} \qquad g^2 \sim k^{-2}$$

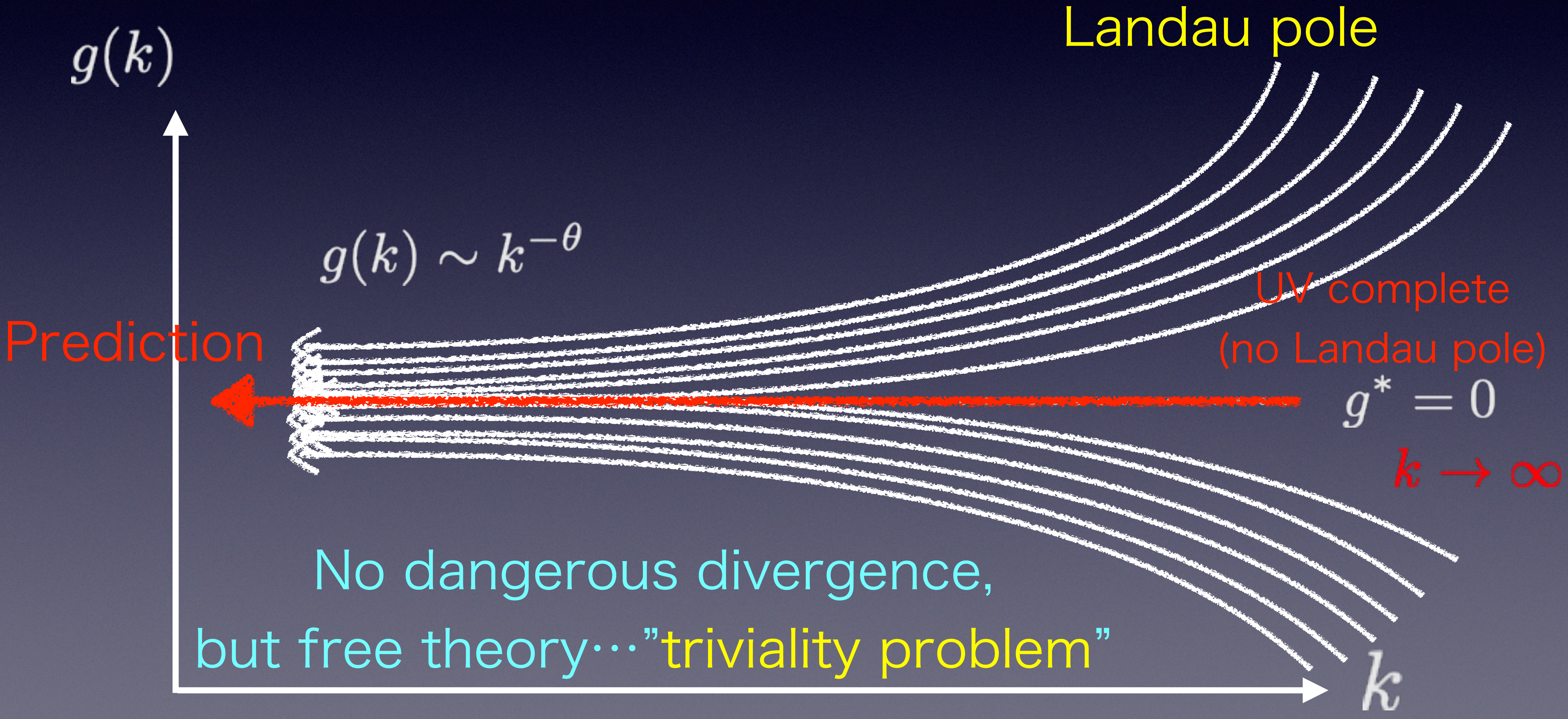
Relevant: $\theta > 0$

- QCD case



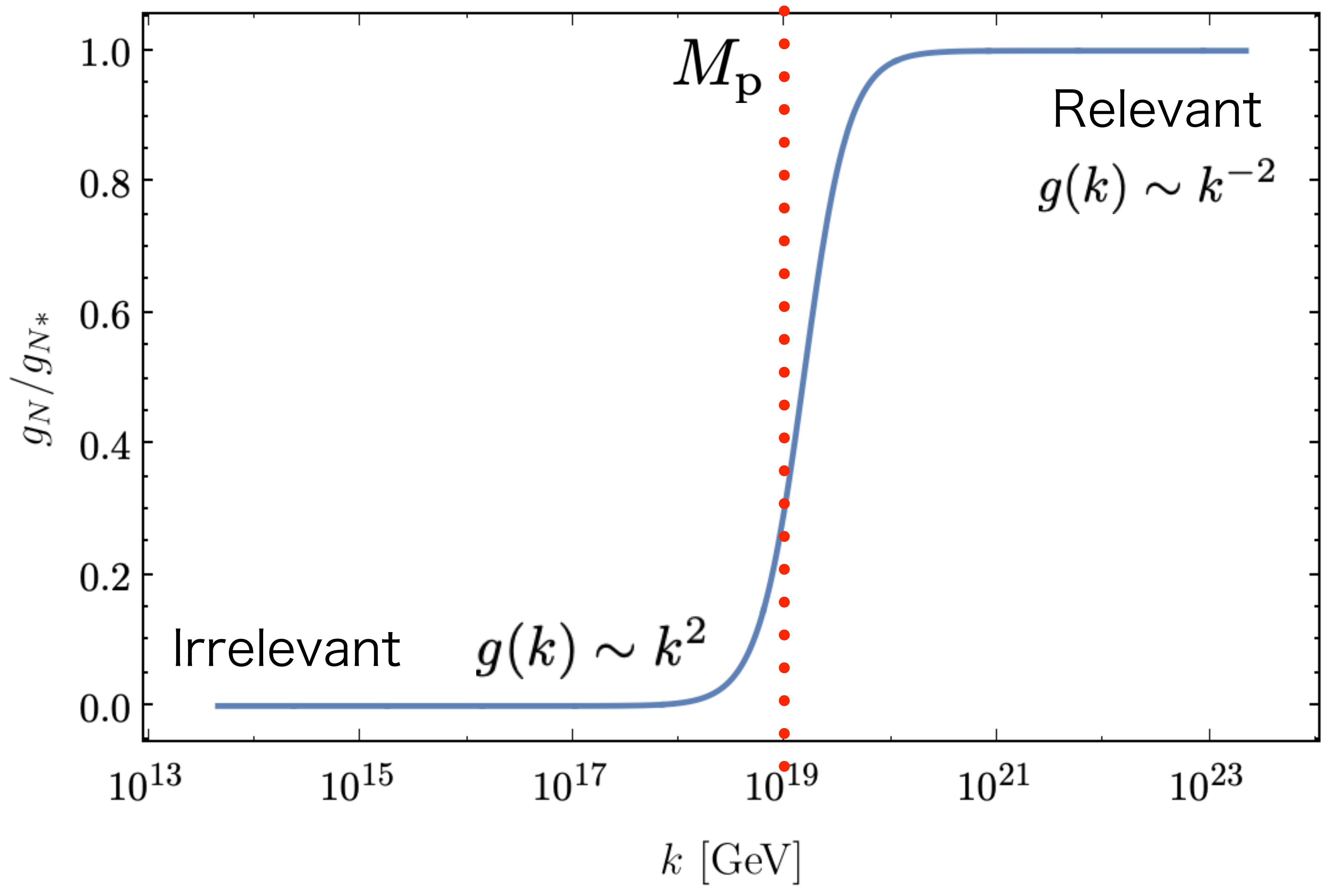
Irrelevant $\theta < 0$

- QED and scalar theory



No dangerous divergence,
but free theory…"triviality problem"

RG flow of Newton constant



Non-trivial FP

$[g_N]$=+2

large anomalous dimension induced

$[g_N]$=-2

Gaussian FP

Degenerate limit

- First-order formalism
  - $e = \det e_\mu{}^a \sim C^4 \to 0$
  - Some components of vierbein degenerate.
  - Admit **topology-change processes** in path integral.

A. Tseytlin,J. Phys. **A15** (1982) L105.
G. T. Horowitz, Class. Quant. Grav. **8** (1991) 587.

- Forbids inverse vierbein
  - Remove terms divergent in the limit $e^\mu{}_a \sim C^{-1} \to \infty$.
  - Do not use inverse metric. $\bar{g}_{\mu\nu} \propto C^2,\ \bar{g}^{\mu\nu} \propto C^{-2}$

Inverse metric

- Metric $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$
  - Canonical normalization $g_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{1}{M_P} h_{\mu\nu}$
- Inverse metric $g_{\mu\alpha} g^{\alpha\nu} = \delta^{\nu}_{\mu}$

$$g^{\mu\nu} = \bar{g}^{\mu\nu} - M_P h^{\mu\nu} + M_P^2 h^{\mu}{}_{\alpha} h^{\alpha\nu} + \cdots$$

Model with null limit

- Including matters, at a certain scale,

$$S = \int \mathrm{d}^4 x \, e \Big[- V + \frac{M^2}{2} e_a{}^{\mu} e_b{}^{\nu} F^{ab}{}_{\mu\nu} - \frac{Z_\psi}{2} \left(\bar{\psi} e_a{}^{\mu} \gamma^a D_\mu \psi + \mathrm{h.c.} \right) \Big]$$

$$D_\mu = \partial_\mu - i g_L (A_\mu)^{ab} \Sigma_{ab} + \cdots$$

- $e = \frac{1}{4!} \epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma} e^a{}_\mu e^b{}_\nu e^c{}_\rho e^d{}_\sigma \sim C^4 \to 0$
- $e e_a{}^{\mu} = \frac{1}{3!} \epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma} e^b{}_\nu e^c{}_\rho e^d{}_\sigma \sim C^3 \to 0$
- $e e_{[a}{}^{\mu} e_{b]}{}^{\nu} = \frac{1}{2!2!} \epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma} e^c{}_\rho e^d{}_\sigma \sim C^2 \to 0$
- $\mathrm{AntiSym}[e e_a{}^{\mu} e_b{}^{\nu} e_c{}^{\rho}] = \epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma} e^d{}_\sigma \propto C \to 0$ No invariant term
- $\mathrm{AntiSym}[e e_a{}^{\mu} e_b{}^{\nu} e_c{}^{\rho} e_d{}^{\sigma}] = \epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma}$ Topological $\epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma} F^{ab}{}_{\mu\nu} F^{cd}{}_{\rho\sigma}$

Degenrate limit

- Higgs potential

$$V(H^\dagger H) = m^2(H^\dagger H) + \lambda(H^\dagger H)^2$$

- In terms of invariance and renormalizability

$$V(H^\dagger H) = \frac{c_1}{H^\dagger H} + \frac{c_2}{(H^\dagger H)^2} + \cdots + m^2(H^\dagger H) + \lambda(H^\dagger H)^2$$

- Inverse terms diverge for H→0 (symmetric phase)

LL gauge theory

- Ordinary YM theory

$$\mathcal{L} = -\frac{1}{4}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(\partial_\mu - igA_\mu^a T^a)\psi$$

T^a commutes with γ^a.

- LL gauge theory

$$\mathcal{L} = -\frac{1}{4}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(\partial_\mu - ig_L A_\mu^{ab}\Sigma_{ab})\psi$$

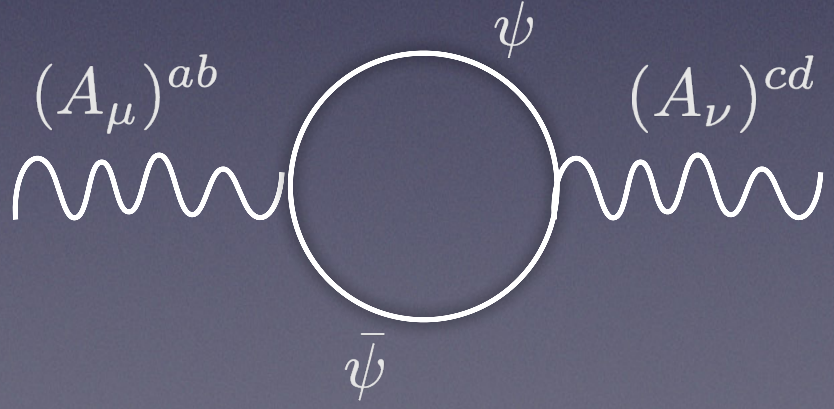
$$\Sigma_{ab} = \frac{1}{4}[\gamma^a, \gamma^b]$$

Σ^{ab} do not commute with γ^a.

Make LL gauge field Dynamical

- The staring action has no kinetic terms except for spinor fields.

- Fermionic fluctuations make other fields dynamical.
    - e.g. LL gauge field in a flat spacetime background



$$= -i\eta^{c[a}\eta^{b]d}\left[f(p^2)\left(\eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) + g(p^2)\frac{p^\mu p^\nu}{p^2}\right]$$

Analogy to QCD

- QCD
  - Nambu-Jona-Lasinio model $\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi - \frac{G}{2}(\bar{\psi}\psi)^2$
  - Quark-meson model $\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi - \frac{m^2}{2}\phi^2 + y\phi\bar{\psi}\psi$

Bosonisation

$\phi \sim \bar{\psi}\psi$

No kinetic term of boson

- Gravity
  - Spinor gravity: Vierbein and LL gauge field are composites of spinors

A. Hebecker, C. Wetterich, Phys.Lett. **B574** (2003) 269-275

  - Our model $S = \int \mathrm{d}^4x\, e\left[-V + \frac{M^2}{2} e_a{}^{\mu} e_b{}^{\nu} F^{ab}{}_{\mu\nu} - \frac{Z_\psi}{2}\left(\bar{\psi} e_a{}^{\mu}\gamma^a D_\mu\psi + \mathrm{h.c.}\right)\right]$

Contents

- Asymptotically safe gravity:
  - non-perturbatively renormalizable quantum gravity
- Universality class at fixed point
  - Duality between asymptotically safe and free theories
- Fermion-induced spacetime

How to formulate?

- Metric theories are diffeomorphism invariant.

$$? \xrightarrow{\text{SSB}} \text{Diffeomorphism}$$

$$g_{\mu\alpha}g^{\alpha\nu} = \delta^{\nu}_{\mu}$$

- In this work, we consider local Lorentz SO(1,3):

$$SO(1,3)_{\text{local}} \times \text{Diff.} \xrightarrow[\cancel{SO(1,3)}_{\text{local}}]{\text{SSB}} \text{Diffeomorphism}$$

$$g_{\mu\alpha}g^{\alpha\nu} = \delta^{\nu}_{\mu}$$

First-order formalism

- Based on SO(1,3) local Lorentz symmetry (and diff.)
  - Vierbein $e_\mu{}^a$
  - Local-Lorentz (LL) gauge field $(A_\mu)^a{}_b$
- Minimal action (Einstein-Hilbert)

$$S = \int \mathrm{d}^4 x\, e \left[-\Lambda + \frac{M^2}{2} e_a{}^\mu e_b{}^\nu F^{ab}{}_{\mu\nu} \right]$$

$$F^a{}_{b\mu\nu} = (\partial A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu])^a{}_b$$

Skippable

First-order formalism

$$S = \int \mathrm{d}^4x\, e\left[-\Lambda + \frac{M^2}{2} e_a{}^{\mu} e_b{}^{\nu} F^{ab}{}_{\mu\nu}\right]$$

- Equation of motion $(A_\mu)^a{}_b = e_\nu{}^a D_\mu e^\nu{}_b$
  - Obtain the EH action in the vierbein formalism
  - Introducing inverse vierbein breaks SO(1,3)$_{\text{local}}$ symmetry.
- Kinetic term of LL gauge field

$$\frac{1}{4} F^{ab}{}_{\mu\nu} F_{ab}{}^{\mu\nu} + \cdots \qquad \to R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \cdots$$

Degenerate limit

- Non-linear σ model: O*(N-1)* invariant
  - Constraint on fields $\langle \phi^i \phi^j \rangle = f_\pi^2 \delta^{ij}$
  - $f_\pi^2 \to 0$: symmetric phase (O*(N)* invariant)
- Gravity in first-order formalism
  - Constrain on metric $g_{\mu\alpha} g^{\alpha\nu} = \delta_\mu^\nu$
  - $C \to 0$: symmetric phase (SO(1,3) invariant).
  - More precisely, $\det(e^a{}_\mu) = 0$

$$\langle e^a{}_\mu \rangle = C \delta^a_\mu$$

$$\bar{g}_{\mu\nu} \propto C^2$$

$$\bar{g}^{\mu\nu} \propto C^{-2}$$

Model with degenerate limit

- Including matters, at a certain scale,

$$S = \int \mathrm{d}^4x\, e \left[-V + \frac{M^2}{2} e_a{}^\mu e_b{}^\nu F^{ab}{}_{\mu\nu} - \frac{Z_\psi}{2} \left(\bar{\psi} e_a{}^\mu \gamma^a D_\mu \psi + \mathrm{h.c.} \right) \right]$$

$$D_\mu = \partial_\mu - i g_L (A_\mu)^{ab} \Sigma_{ab} + \cdots$$

  - Invariant under SO(1,3)$_{\text{local}}$ × diff.
  - Only fermions are dynamical!
  - No kinetic terms of vierbein, gauge fields, scalar fields.
  - These fields would be dynamical via fermion quantum corrections.

Spontaneous local Lorentz symmetry breaking

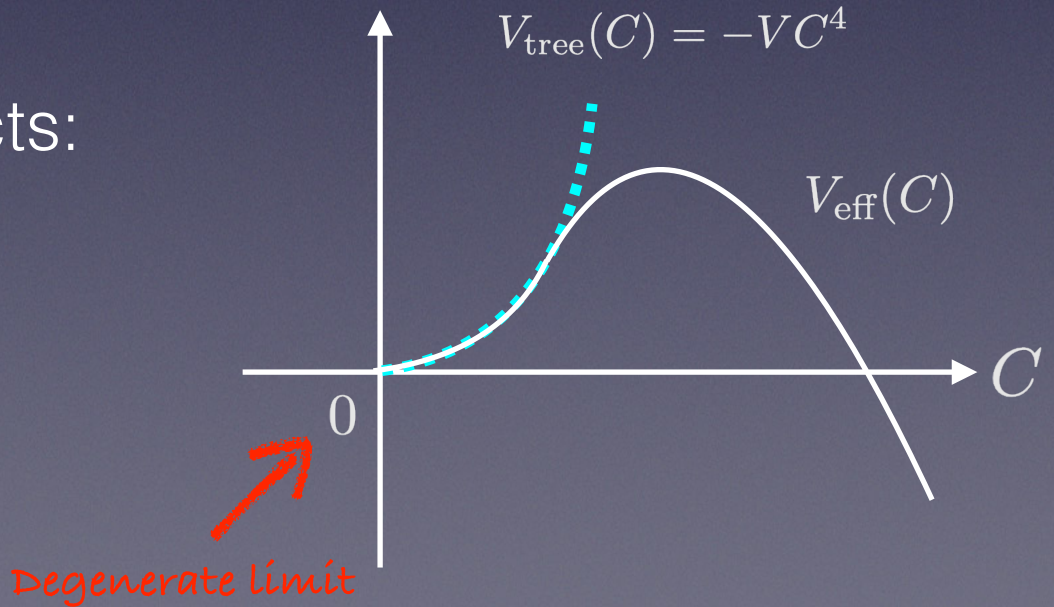
- $SO(1,3)_{local} \times$ diff.
- Generation of expectation value of vierbein
- A possible solution would be a flat spacetime.

$$\langle e^a{}_\mu \rangle = C\delta^a_\mu$$

- Effective potential from spinor loop effects:

$$V_{\rm eff}(C) = -VC^4 - \frac{(CM)^4}{2(4\pi)^2}\log\left(C^2M^2/\mu^2\right)$$

- Precise analysis is in progress.



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Spontaneous local Lorentz symmetry breaking

- Local Lorentz gauge symmetry is broken.
  - Degrees of freedom (d.o.f.):

    Symmetric part (metric)
    (radial modes)

    Anti-symmetric part (torsion)
    (NG modes)

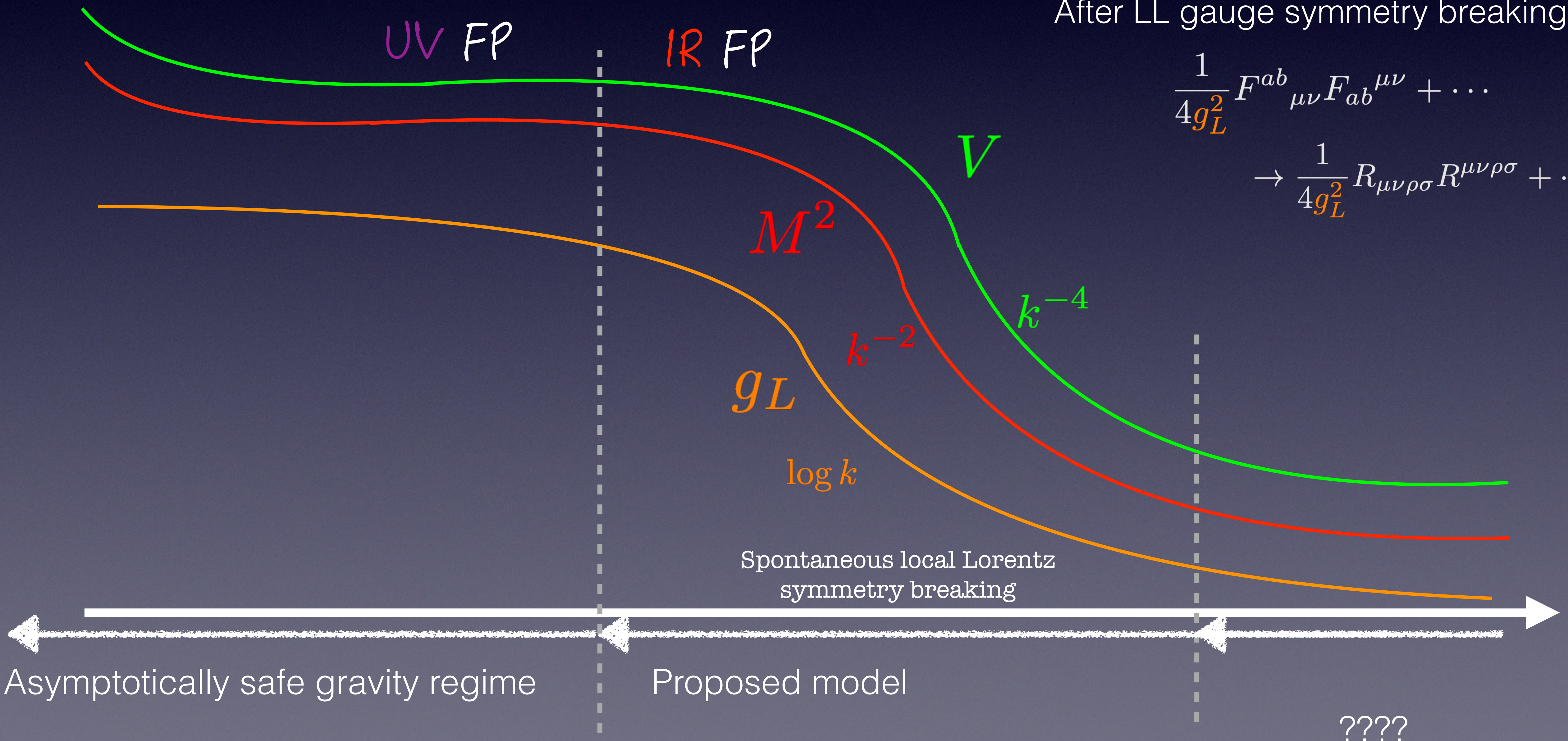
    - Vierbein $e_\mu{}^a$: 16 d.o.f. = 10 + 6 d.o.f.

      eaten

    - LL gauge field $(A_\mu)^a{}_b$: 6 d.o.f.
  - LL gauge bosons become massive and decouple.
  - The symmetry parts (radial modes) are still massless thanks to diif..

RG flow (ideal)

$$S = \int \mathrm{d}^4x\, e\left[-V + \frac{M^2}{2} e_a{}^{\mu} e_b{}^{\nu} F^{ab}{}_{\mu\nu} - \frac{Z_\psi}{2}\left(\bar{\psi} e_a{}^{\mu}\gamma^a(\partial_\mu - i g_L A_\mu)\psi + \mathrm{h.c.}\right)\right]$$



After LL gauge symmetry breaking

$$\frac{1}{4g_L^2} F^{ab}{}_{\mu\nu} F_{ab}{}^{\mu\nu} + \cdots$$

$$\to \frac{1}{4g_L^2} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \cdots$$

Spontaneous local Lorentz symmetry breaking

Asymptotically safe gravity regime

Proposed model

????

$SO(1,3)_{\text{global}}$ × diff.

$SO(1,3)_{\text{local}}$ × diff.